

A Generalization of the Classical Kelly Betting Formula to the Case of Temporal Correlation

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Outline of Today's Talk

- What is **Kelly betting** + its **limitations**?
- **Temporal correlations** between plays – **history-cognizant** coin
- New generalized framework for temporally correlated Kelly betting
- **Simulation studies** to highlight the **advantages** offered
- **Conclusions** and directions for **future research** and **applications**

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Kelly Betting – Why?

- Let $X_k \in \{-1, 1\}$ be the return on **heads/tails** (coin flip) or **up/down tick** (stock price) - seminal paper of Kelly (1956)
- The question: if $\mathbb{P}(X_k = 1) = p > 1/2$, what **fraction K** of account value V_k , should be **bet on $X_k = 1$** ?
- Maximizing the expected value of V_{k+1}

$$E(V_{k+1}) = p(1 + K)V_k + (1 - p)(1 - K)V_k$$

we obtain $K^* = 1 - \text{bet the farm!}$ But... what if we **lose once**?

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Kelly Betting – How?

- Kelly proposed optimizing the **Expected Logarithmic Growth (ELG)** of account value

- Suppose a sequence of n i.i.d. plays

$$\mathbf{X} = \{X_0, X_1, \dots, X_{n-1}\} \in \{-1, 1\}^n$$

- The resulting account value is

$$V_n(\mathbf{X}, K) = V_0 \prod_{k=0}^{n-1} [1 + KX_k]$$

- $\text{ELG}(K) = \frac{1}{n} \mathbb{E} \left\{ \log \left[\frac{V_n(\mathbf{X}, K)}{V_0} \right] \right\}$ – maximized at $K^* = 2p - 1$

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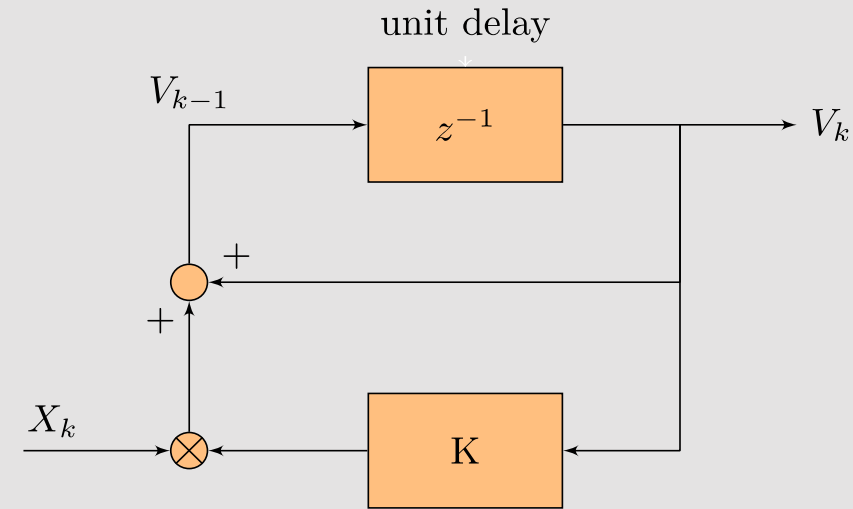
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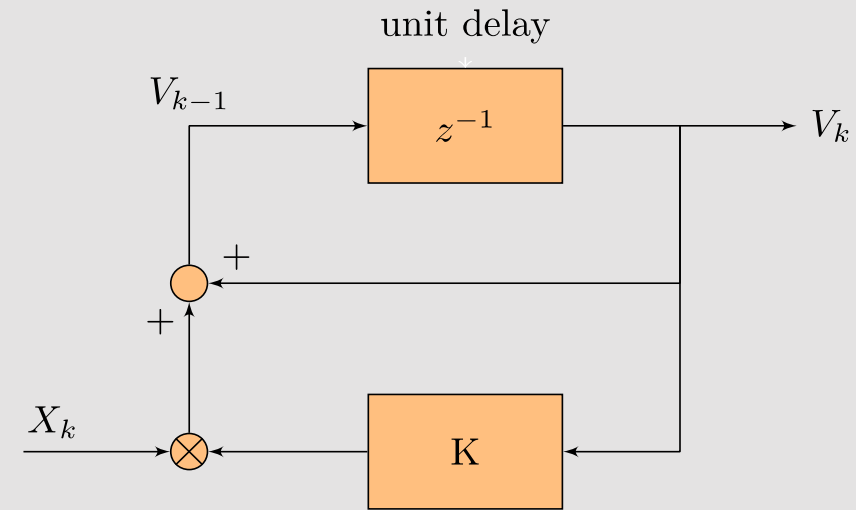
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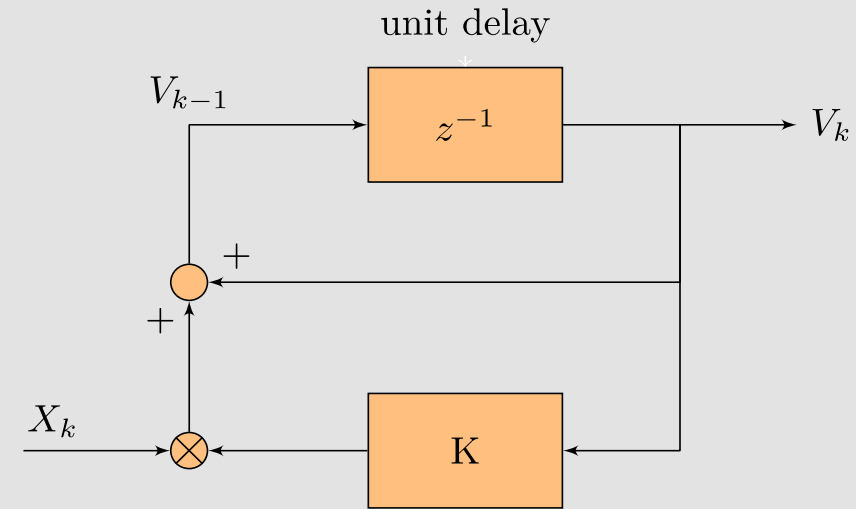
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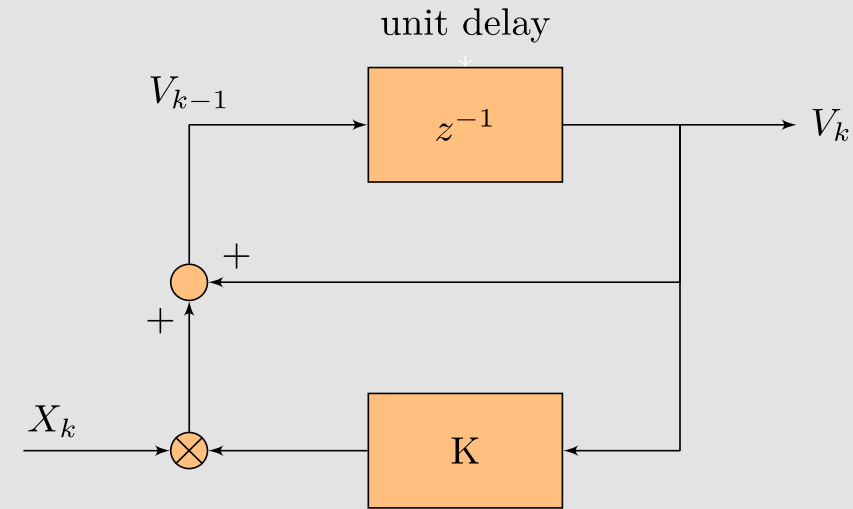
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Kelly Betting & Limitations

- Kelly's theory has been extensively pursued in the literature – Rotando & Thorp (1992); MacLean, Thorp, & Ziemba (2010); ...
- Including **extensions** and **analysis of asymptotic properties** amongst many other areas – Thorp (2006); Hsieh, Barmish, & Gubner (2016); Busseti, Ryu, & Boyd (2016); ...
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Temporally Correlated Plays

- Classically, the literature assumes the X_k are i.i.d., whereas we study the probability of a sequence \mathbf{X} is given by

$$P_{\mathbf{X}} = \prod_{k=0}^{n-1} \mathbb{P}(X_k | X_{k-1}, X_{k-2}, \dots, X_{k-m})$$

- Where the **memory depth** is denoted by m
- Our first contribution is the **generalized optimal betting fraction**

$$K_n = 2 \left(\frac{\mathbb{E}[H_n]}{n} \right) - 1$$

- $\mathbb{E}[H_n]$ - expected number of heads in seq. of length n

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History-cognizant Coin with $m = 1$

- Consider a **one-memory coin** such that

$$\mathbb{P}(X_k = 1 \mid X_{k-1}) = \omega_0 + \omega_1 X_{k-1}$$

- It is shown in this case that

$$\frac{\mathbb{E}[H_n]}{n} = \lambda_n p_0 + (1 - \lambda_n) p_\infty$$

- Where

$$\lambda_n = \frac{1}{n} \left[\frac{1 - (2\omega_1)^n}{1 - 2\omega_1} \right]; \quad p_0 = \omega_0 + \omega_1 x_{-1};$$
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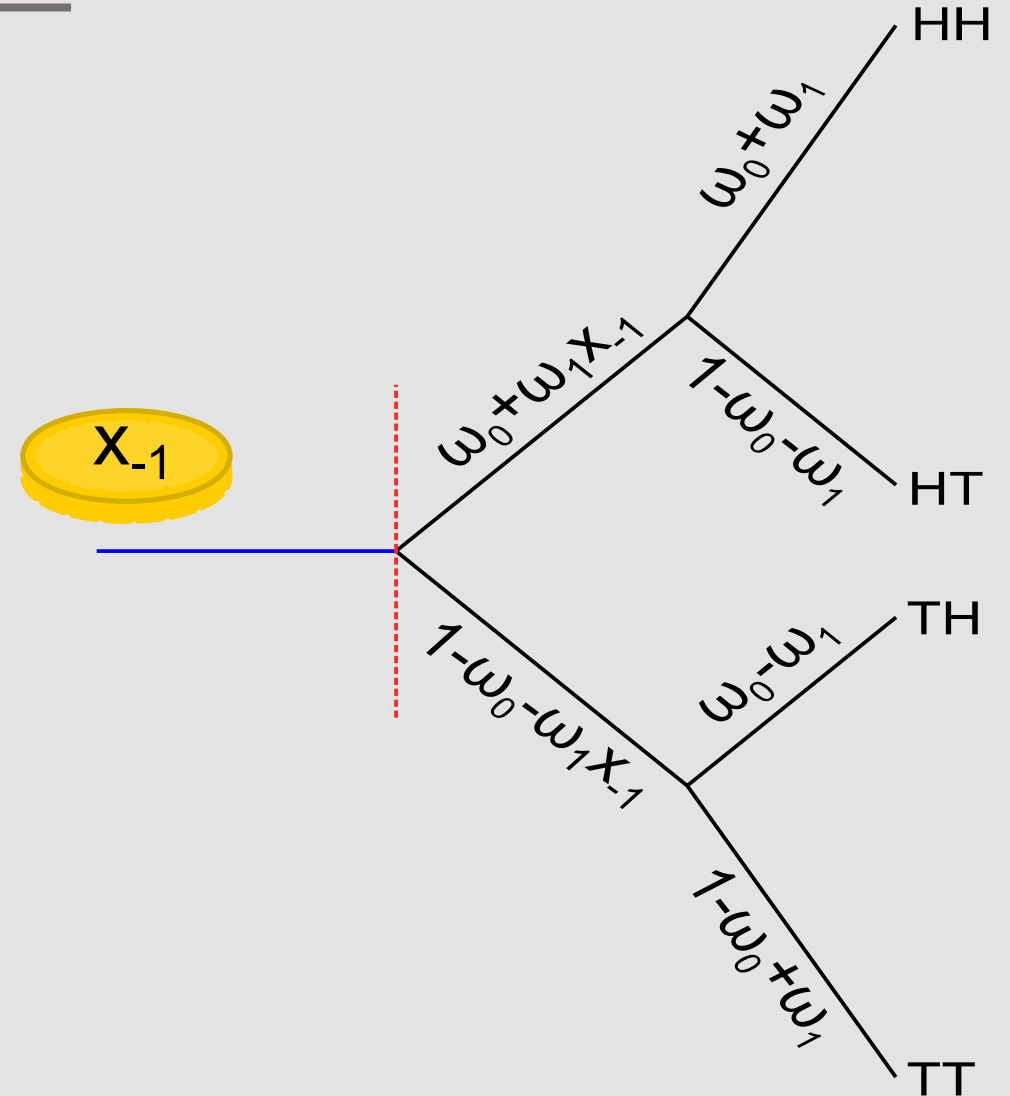
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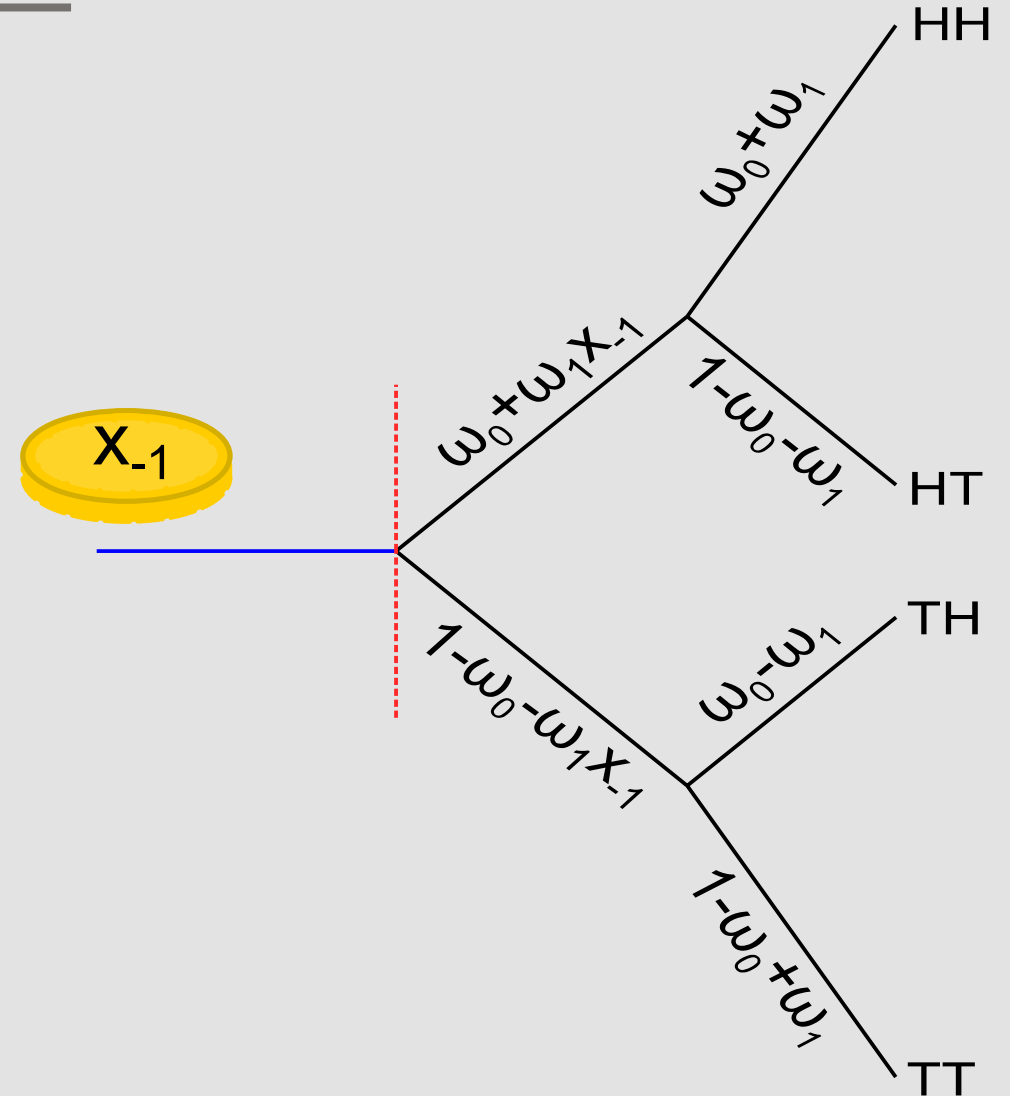
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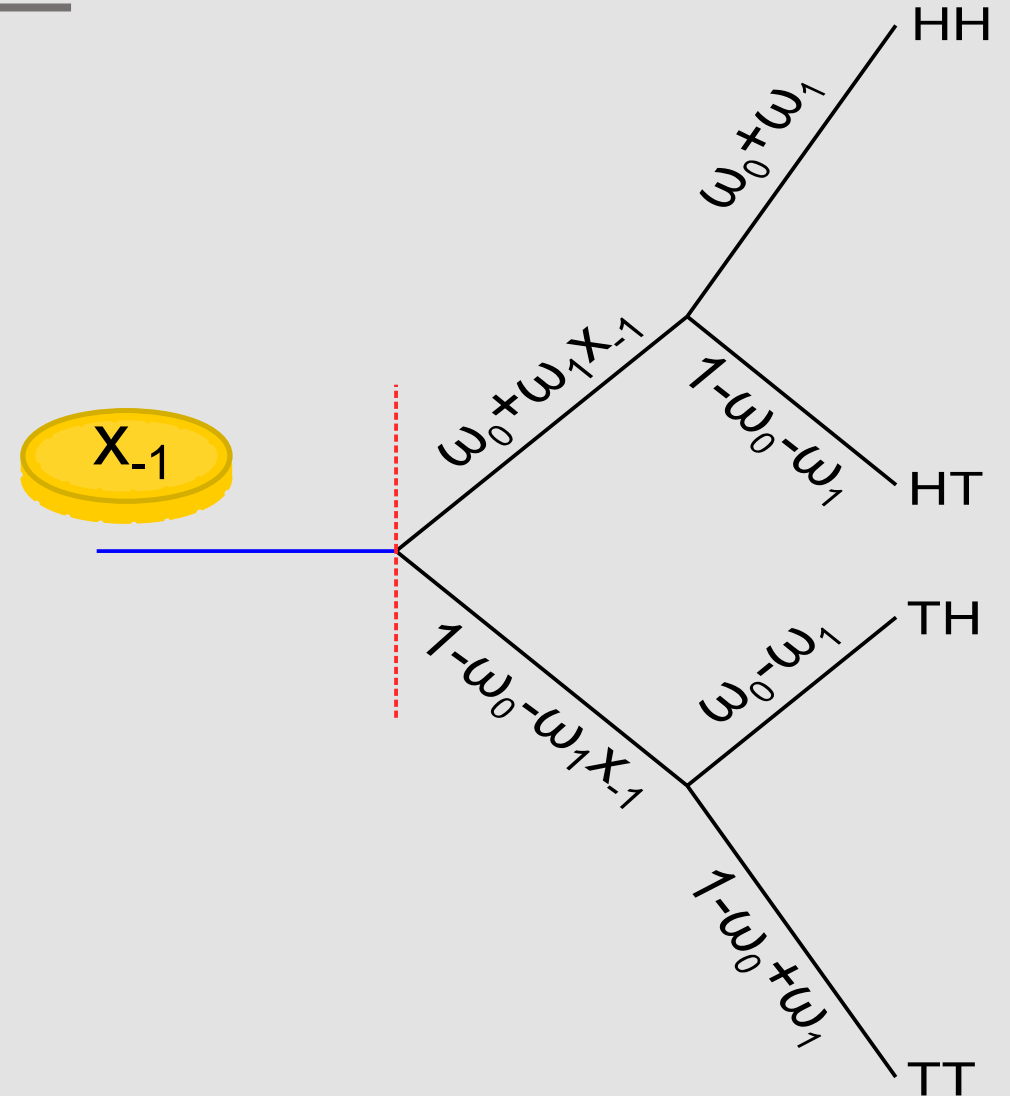
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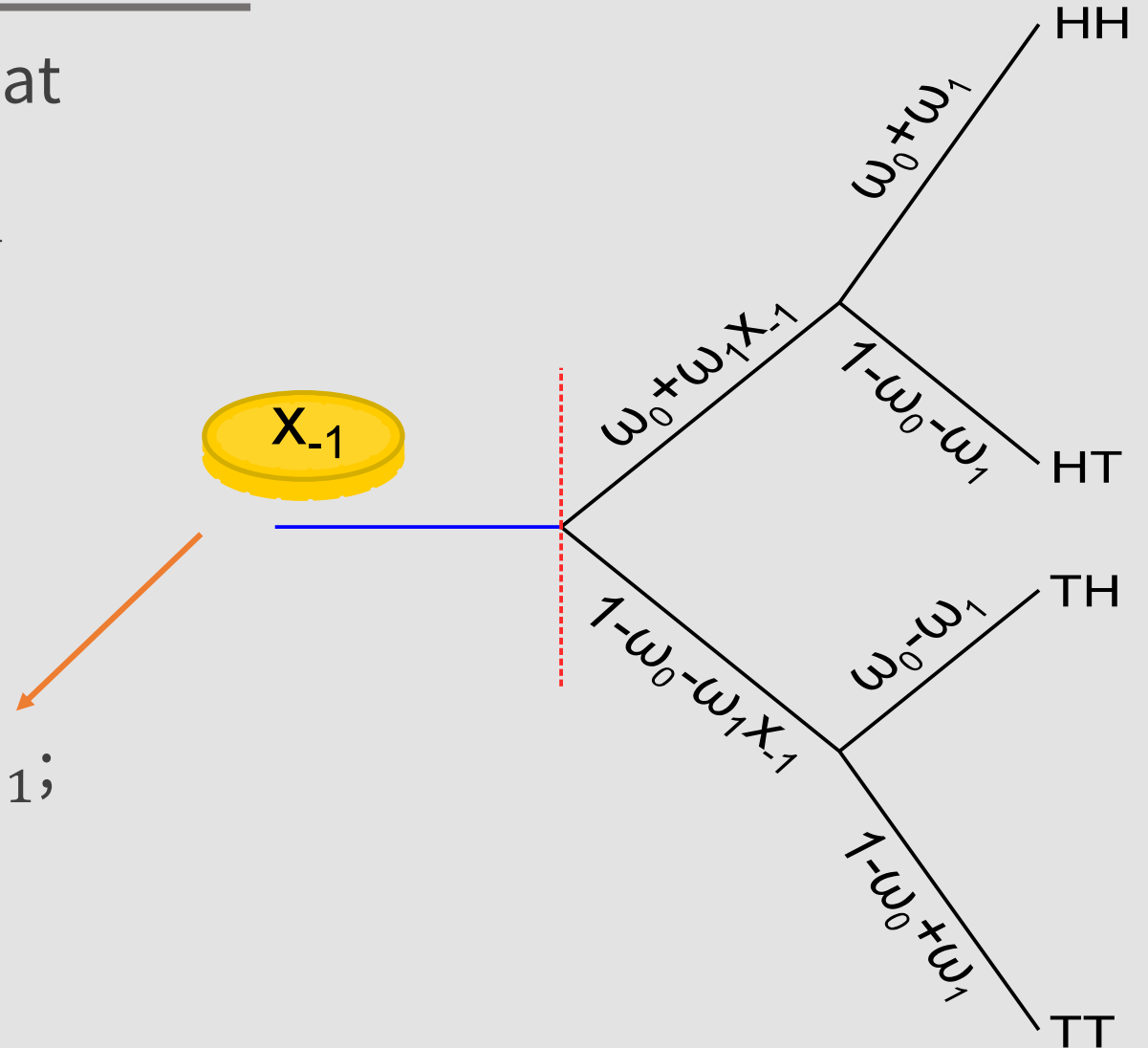
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History-cognizant Coin with $m > 1$

- General linear m -memory coin:

$$p_k = \mathbb{P}(X_k = 1 \mid X_{k-1}, X_{k-2}, \dots, X_{k-m}) = \omega_0 + \sum_{i=1}^m \omega_i X_{k-i}$$

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$$\mathbb{E}[H_n] = \sum_{k=0}^{n-1} p_k$$

- Where

$$p_k = \omega_0 - \sum_{i=1}^m \omega_i + 2 \sum_{i=1}^m \omega_i p_{k-i}$$

- Allowing K_n for **arbitrary memory length** to be obtained!

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Further Results

- 1. Multiple outcomes:** $x_1, x_2, \dots, x_\ell \in (-1, \infty)$ (different stock price changes,...), with arbitrary P_X

$$\text{ELG}(K) = \sum_{i=1}^{\ell} \frac{\mathbb{E}(H_{n,i}(X))}{n} \log(1 + Kx_i)$$

where $H_{n,i}(X)$ describes no. times $X_k = x_i$ in a seq. of length n

- 2. Time-varying feedback:** $K = [\tilde{K}_0, \tilde{K}_1, \dots, \tilde{K}_{n-1}]$ it can be shown that the optimal betting fraction is

$$\tilde{K}_k = 2p_k - 1$$

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Simulation Study - I

- Simulate sequences of length n in the case of the one-memory coin
- Three bettors:
 1. A classical K^* -bettor who uses the standard Kelly result
 2. The K_n -bettor who uses a (fixed) fraction incorporating the autocorrelations
 3. Lastly, a \mathbf{K} -bettor using a varying betting fraction for each bet

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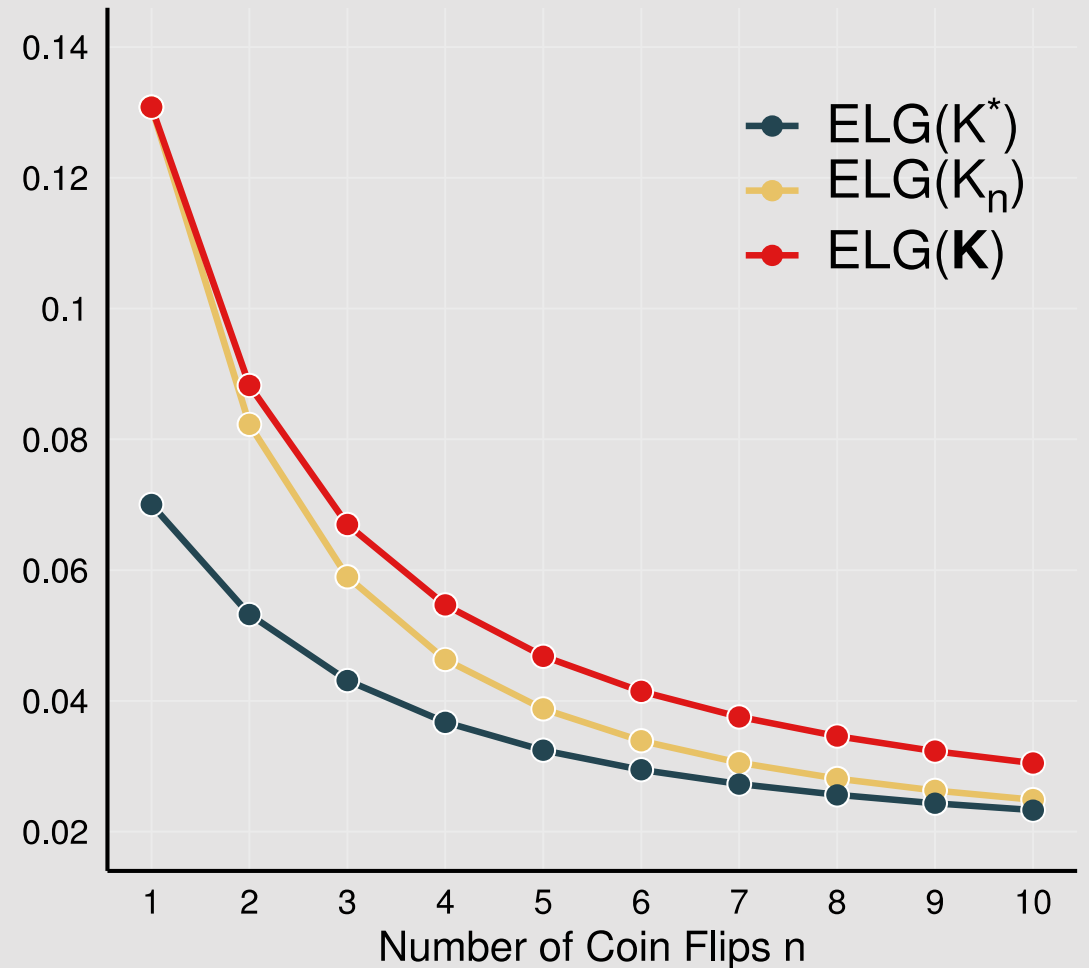
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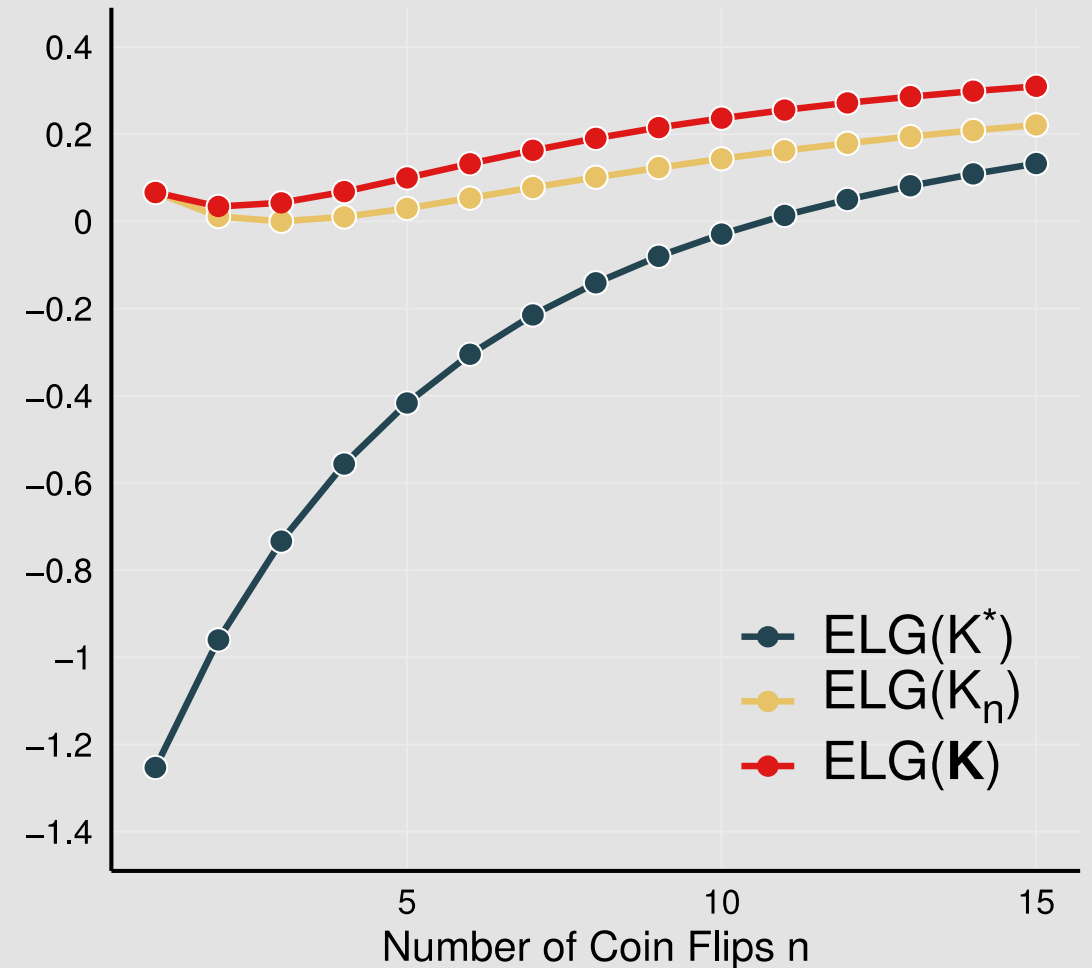
(a) $\omega_0 = 0.55$, $\omega_1 = 0.2$



Simulation Study - II

- Simulate sequences of length n in the case of the one-memory coin
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(c) $\omega_0 = 0.35$, $\omega_1 = 0.33$



Conclusions & Future Work

- **Generalized framework** for incorporating **temporal autocorrelation**
- Obtain, for the **history-cognizant coin**, closed form solutions for the **optimal betting fraction**
- Extension **beyond the two-outcome game** to multiple outcomes and additionally incorporate a **time-varying betting fraction**
- Advantages demonstrated via **numerical simulations**
- **Future work** – Application of the theory to **algorithmic trading**, extend theory for correlations **across multiple assets**, ...

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Collaborators & Thanks




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A Generalization of the Classical Kelly Betting Formula to the Case of Temporal Correlation

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Abstract—For sequential betting games, Kelly's theory, aimed at maximization of the logarithmic growth of one's account value, involves optimization of the so-called betting fraction K . In this letter, we extend the classical formulation to allow for temporal correlation among bets. To demonstrate the potential of this new paradigm, for simplicity of exposition, we mainly address the case of a coin-flipping game with even-money payoff. To this end, we solve a problem with *memory depth* m . By this, we mean that the outcomes of coin flips are no longer assumed to be i.i.d. random variables. Instead, the probability of heads on flip k depends on previous flips $k-1, k-2, \dots, k-m$. For the simplest case of n flips, with $m=1$, we obtain a closed form solution K_n for the optimal betting fraction. This generalizes the classical result for the mem-

gain. That is, with V_k being the account value after k plays, the optimal $(k+1)$ -th bet size is K^*V_k , where, for classical coin flipping with even-money payoff, $K^* = 2p - 1$.

The ELG approach has resulted in a voluminous body of literature extending and applying the theory to other well-known gambling games such as Blackjack and sports betting considered in [2] and [3], and asset management and stock trading as in [4]–[10]; see also the extensive bibliography in the textbook [11]. Papers such as [12]–[17] have also covered related issues including asymptotics, problems related to aggressiveness of wagers and alternative risk metrics.

The main feature which differentiates this letter from exist-

Thank you for listening!

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