Branching Processes and their Application to Popularity Dynamics **Two Short Stories**

Joseph O'Brien

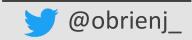
5 - Nov - 2020 | naXys Seminar Series, Namur, Belgium





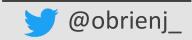


- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model diffusion on social media.
- What they offer in terms of **predictability** of cascades.
- Conclusions.



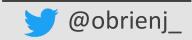
- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model diffusion on social media.
- What they offer in terms of **predictability** of cascades.

• Conclusions.



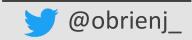
- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model **diffusion on social media**.
- What they offer in terms of **predictability** of cascades.

• Conclusions.

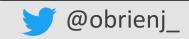


- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model **diffusion on social media**.
- What they offer in terms of **predictability** of cascades.

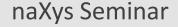
• Conclusions.

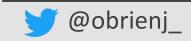


- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model **diffusion on social media**.
- What they offer in terms of **predictability** of cascades.
- Conclusions.



- What is a **branching process**?
- Modelling **random-copying** mechanisms.
- How they can be used to model **diffusion on social media**.
- What they offer in terms of **predictability** of cascades.
- Conclusions.





• **Francis Galton** proposed a formulation to understand the likelihood of a family name staying alive in Educational Times 1873 through the following question

PROBLEM 4001: A large nation, of whom we will only concern ourselves with adult males, N in number, and who each bear separate surnames colonise a district. Their law of population is such that, in each generation, a_0 percent of the adult males have no male children who reach adult life; a_1 have one such male child; a_2 have two; and so on up to a_5 who have five. Find (1) what proportion of their surnames will have become extinct after r generations; and (2) how many instances there will be of the surname being held by m persons."

138 WATSON and GALTON.-Extinction of Families.

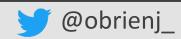
Mr. Galton then read the following paper by the Rev. H. W. Watson and himself:

On the PROBABILITY of the EXTINCTION of FAMILIES. By the Rev. H. W. WATSON. With PREFATORY REMARKS, by FRANCIS GALTON, F.R.S.

THE decay of the families of men who occupied conspicuous po-

Rev H. W. Watson



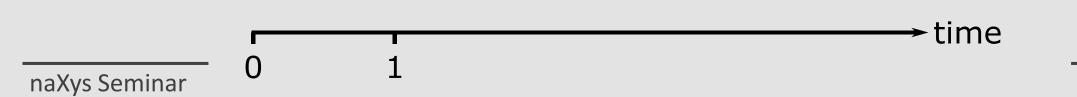


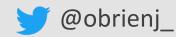


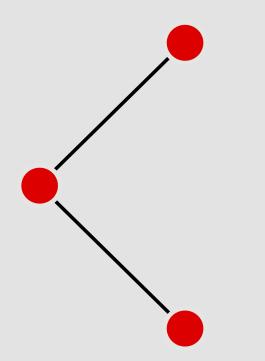
0

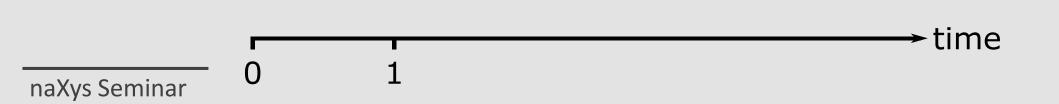
→ time

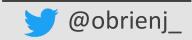
🈏 @obrienj_

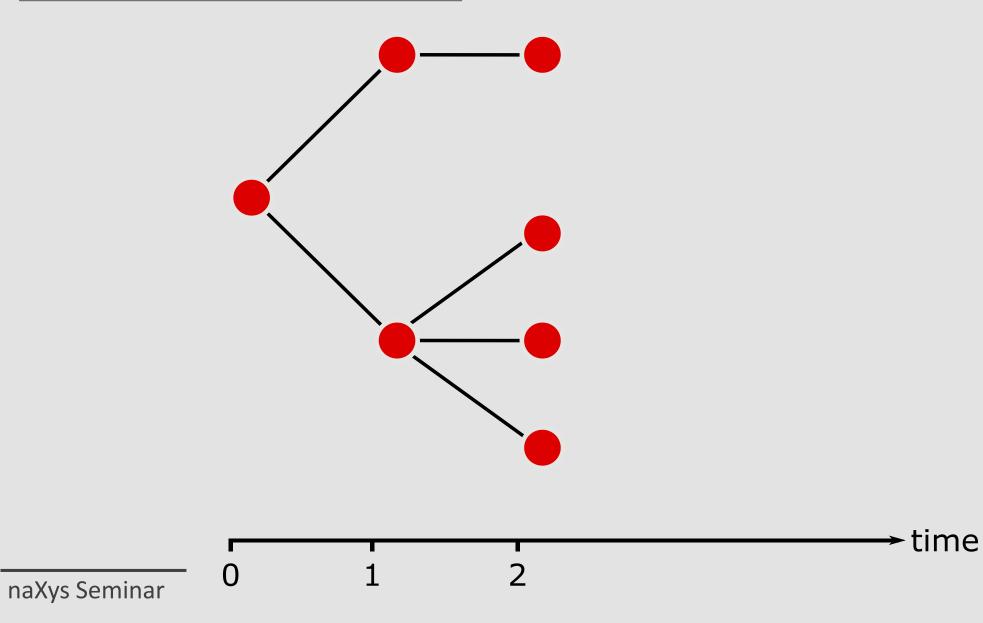


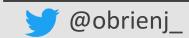


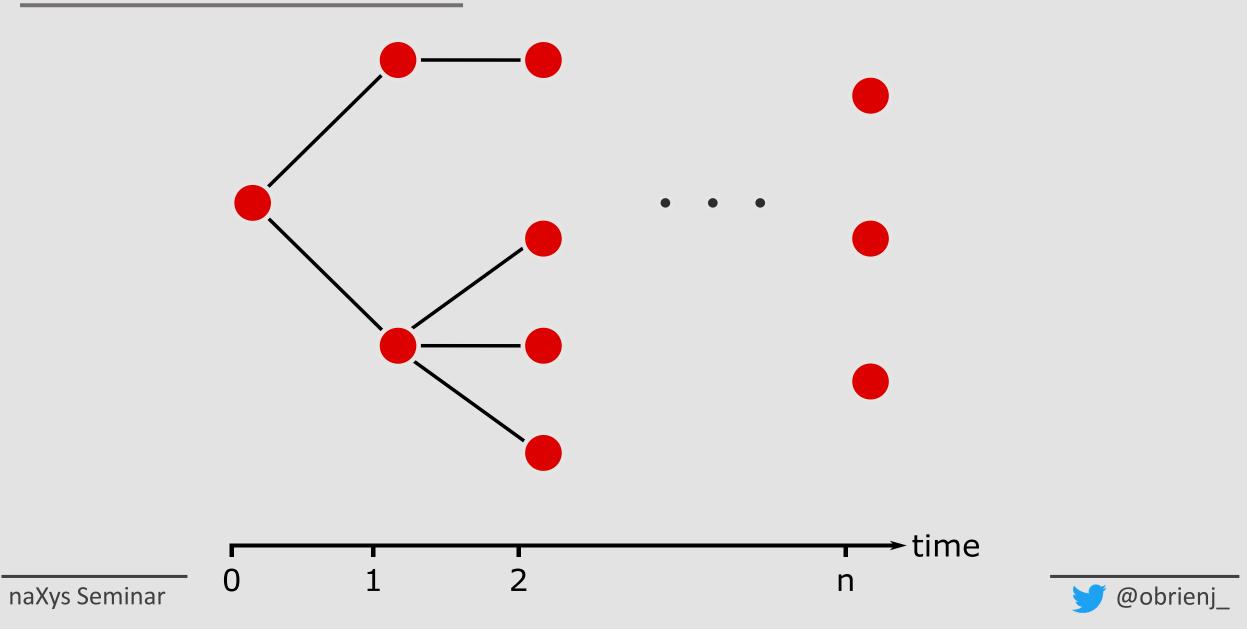


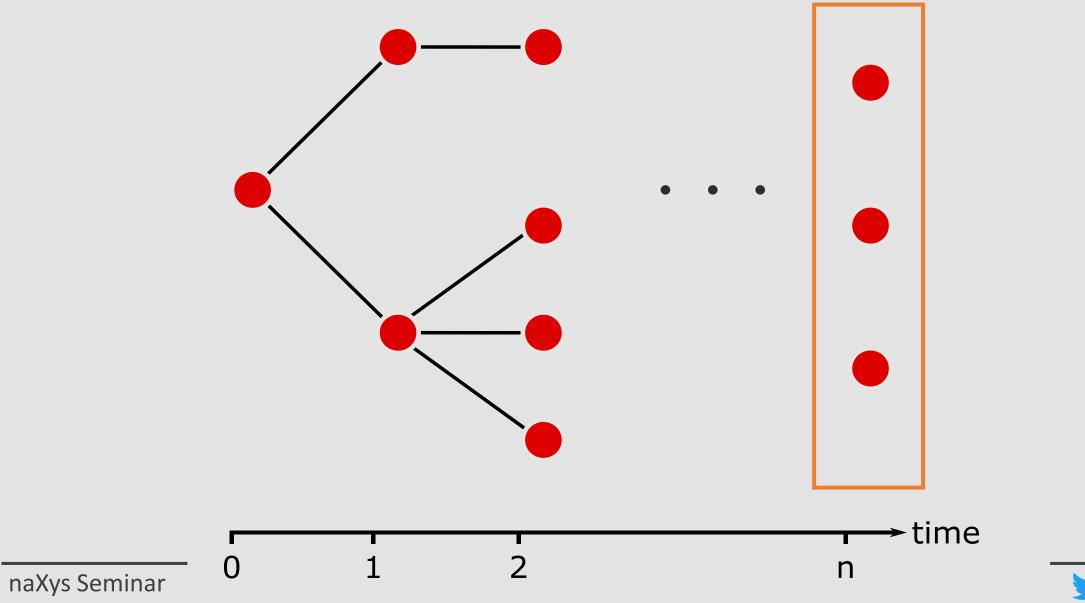




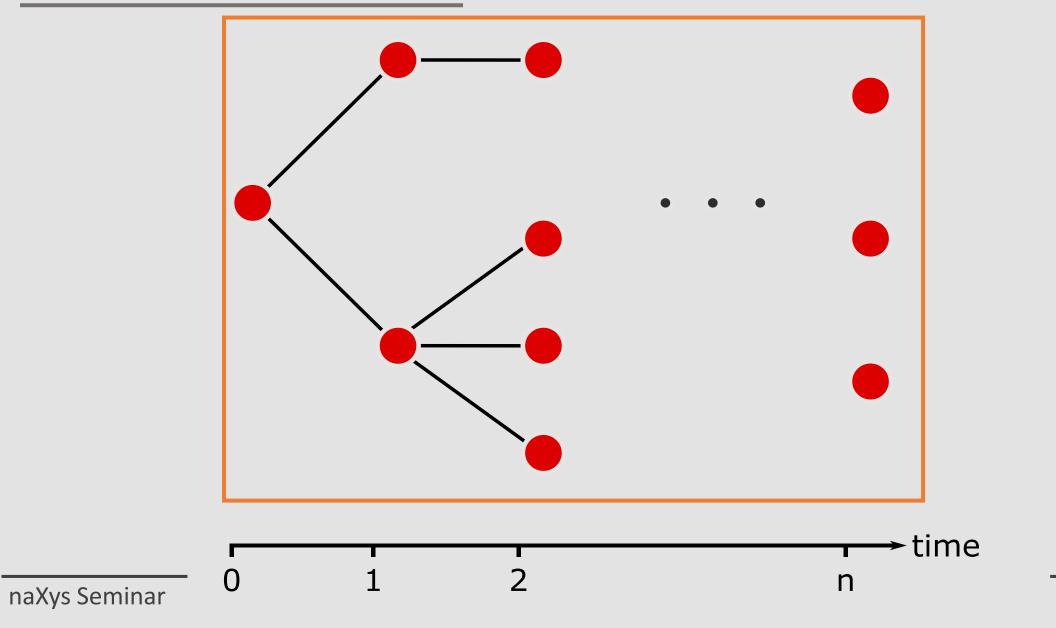


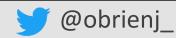


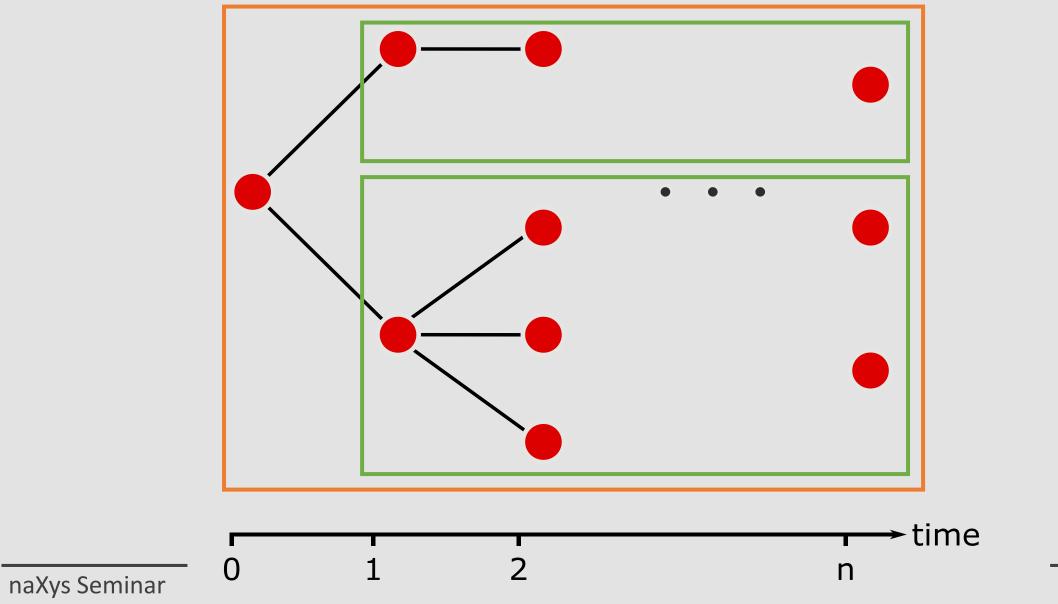


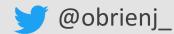












While classically used for biological purposes – recently have been used to describe popularity dynamics including:

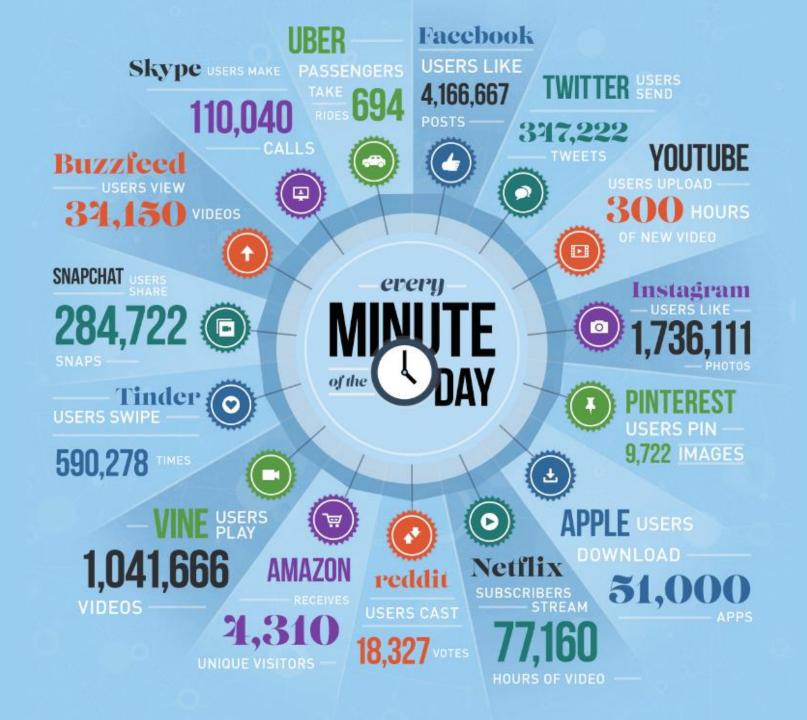
Citation dynamics

- M. V. Simkin and V. P. Roychowdhury, *J. Am. Soc. Inf. Sci. Tech.* 58, 1661 (2007). Viral marketing
- R. Van der Lans, et al., *Market. Sci.* 29, 348 (2010).
- J. L. Iribarren and E. Moro, *Phys. Rev. E* 84, 046116 (2011).

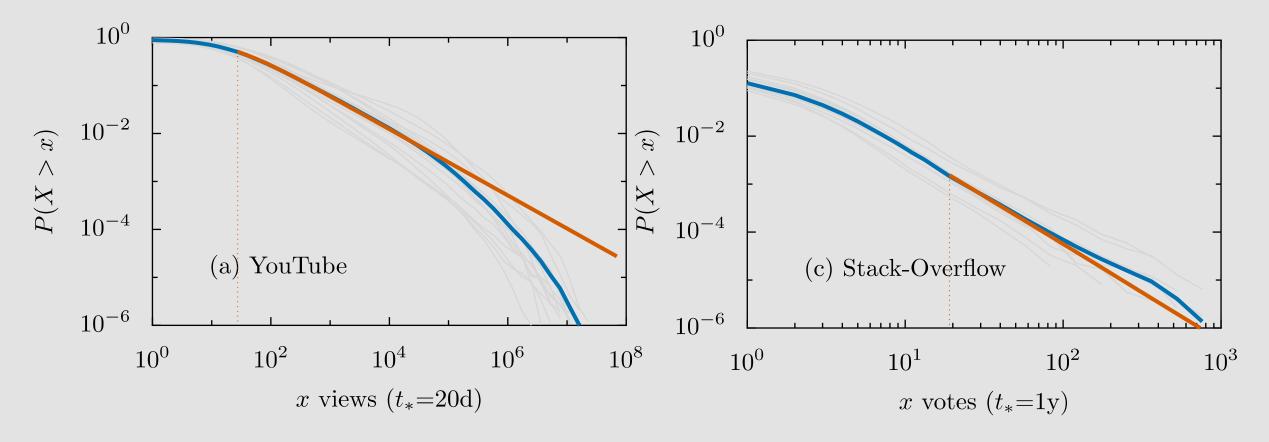
Social media cascades

- O. Yagan at al., *IEEE J. Sel. Areas Commun.* 31, 1038 (2013).
- J. P. Gleeson et al., *Phys. Rev. Lett.* 112, 048701 (2014).
- J. P. Gleeson at al., *Phys. Rev. X* 6, 021019 (2016).

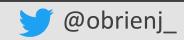




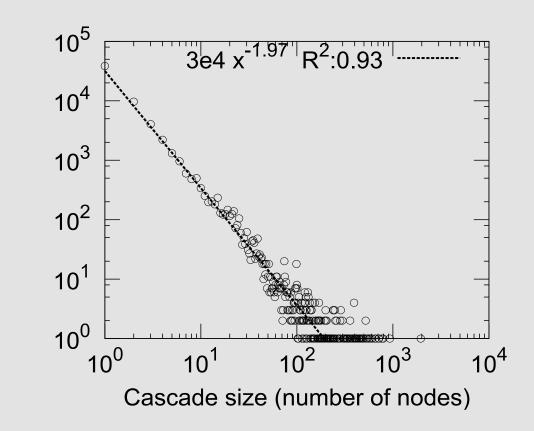
Empirical cascades



J. M. Miotto, E. G. Altmann, Predictability of Extreme Events in Social Media. *PLoS ONE* 9(11) (2014)

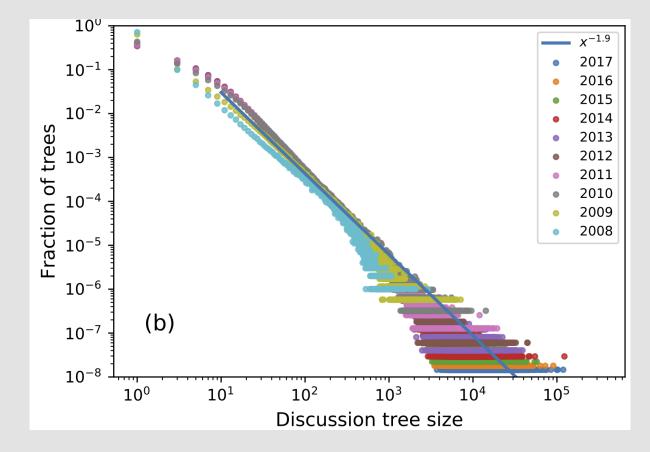


Empirical cascades

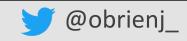


J. Leskovec et al. "Patterns of cascading behavior in large blog graphs."

Proc. SIAM Intern. Conf. on Data Mining (2007).



A. N. Medvedev, R. Lambiotte, and J.-C. Delvenne. "The anatomy of Reddit: An overview of academic research." *Dyn. on and of Comp. Net.* Springer, Cham, 2017(2017).



Count

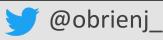
Empirical cascades

MULTI-NETWORKING

Average number of social media accounts held by internet users

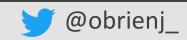


Dave Chaey. Global social media research summary 2017. *Smart Insights: Social Media Marketing*, 2018.



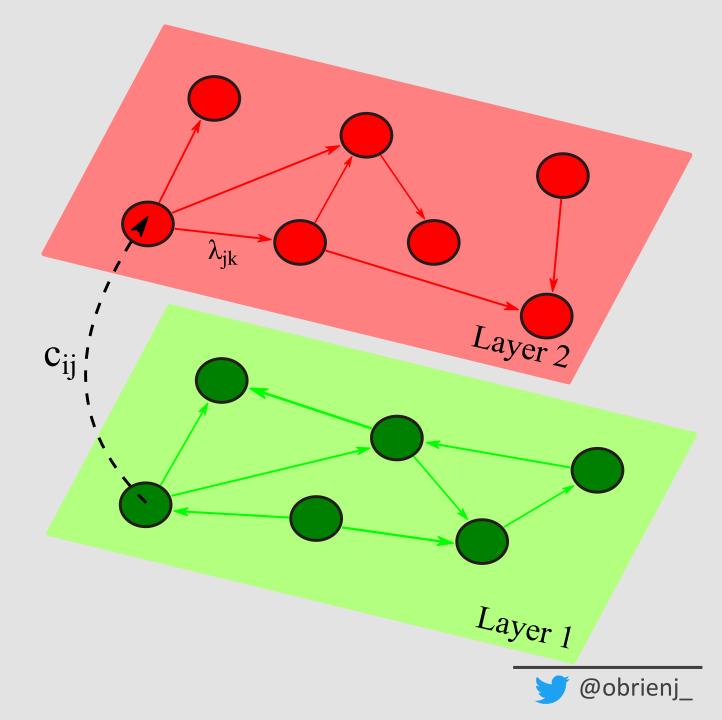
What we want to obtain

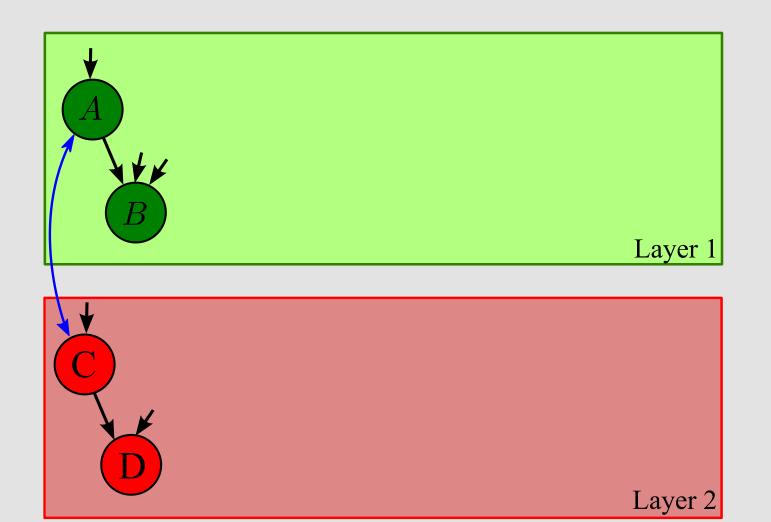
- A model which can capture the **features of empirical data**.
- Uses the **exact network** topology (rather than ensembles of random graphs).
- Considers the fact that different **users** have **different behaviours**.
- Allows users to have accounts on **multiple platforms**.



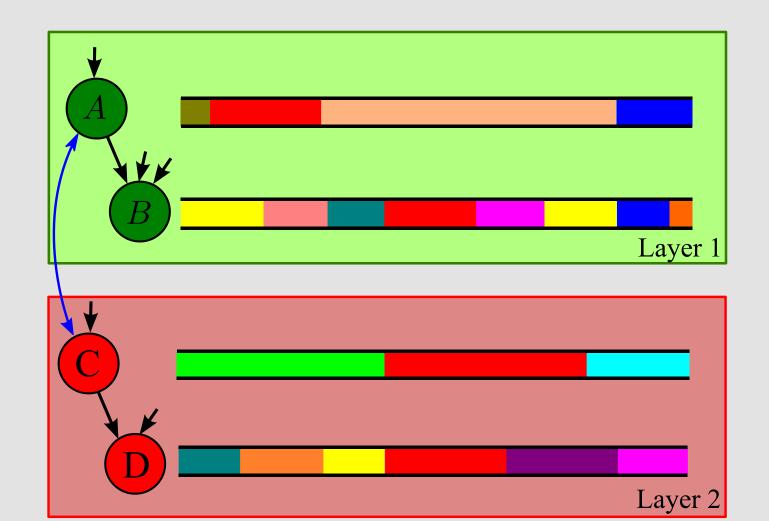
What we want to obtain

- A model which can capture the **features of empirical data**.
- Uses the **exact network** topology (rather than ensembles of random graphs).
- Considers the fact that different **users** have **different behaviours**.
- Allows users to have accounts on **multiple platforms**.

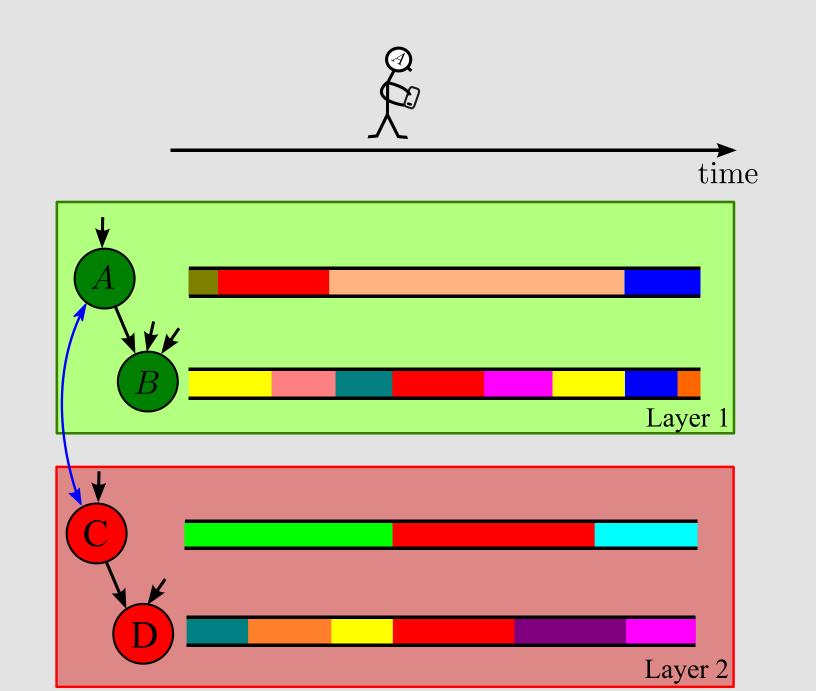




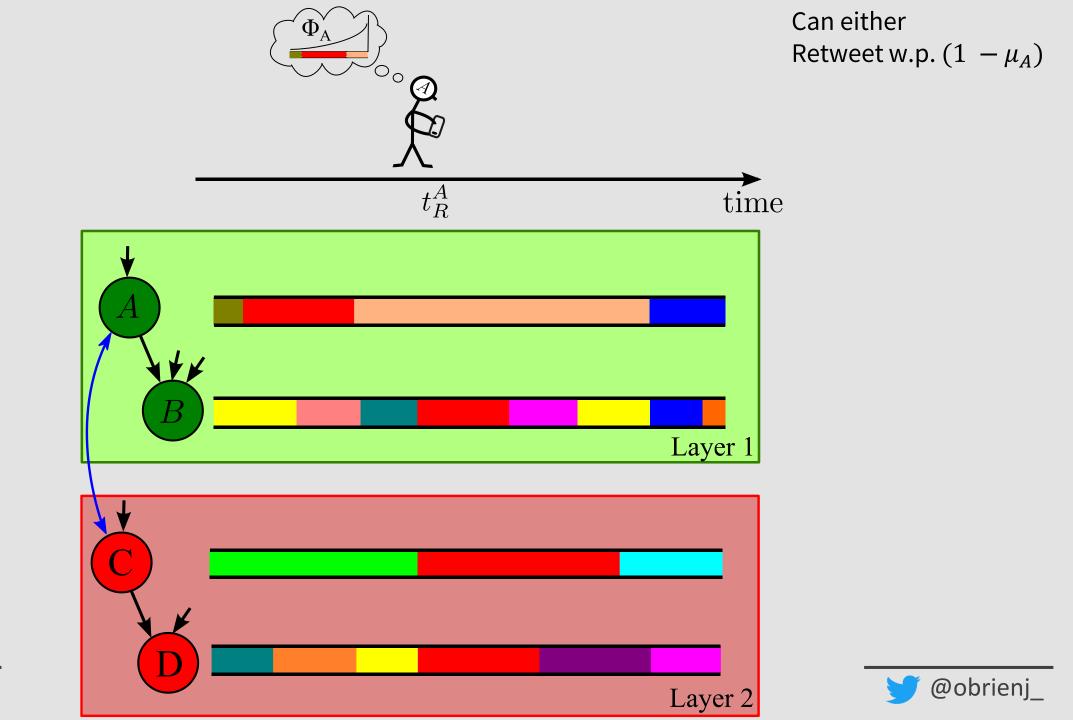


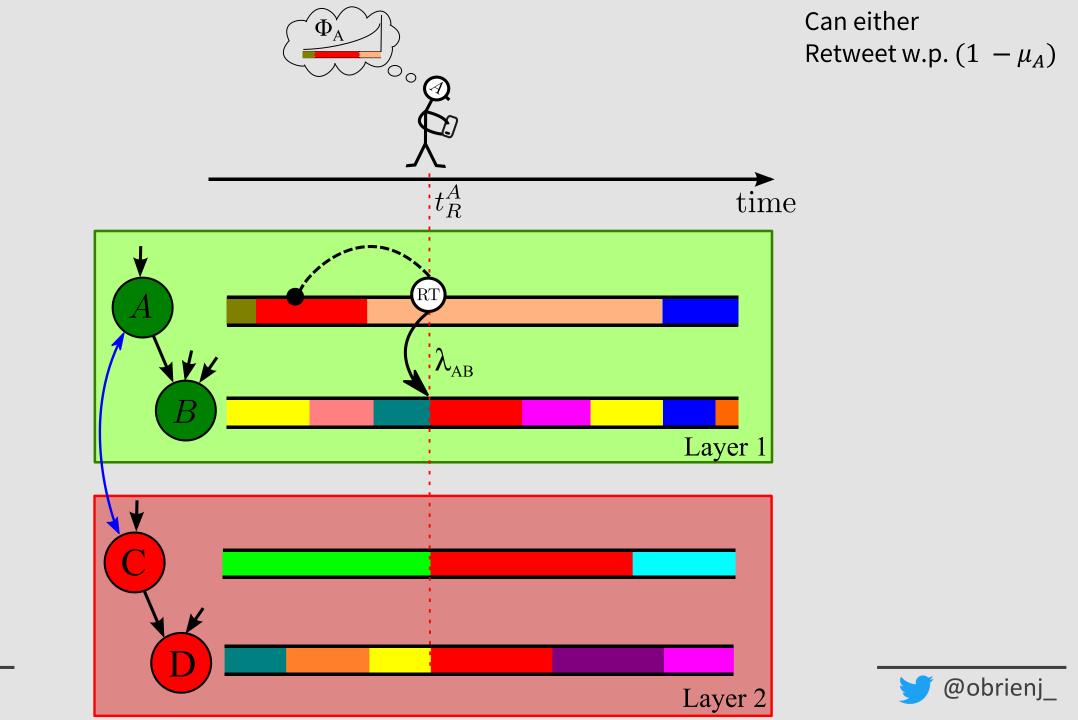


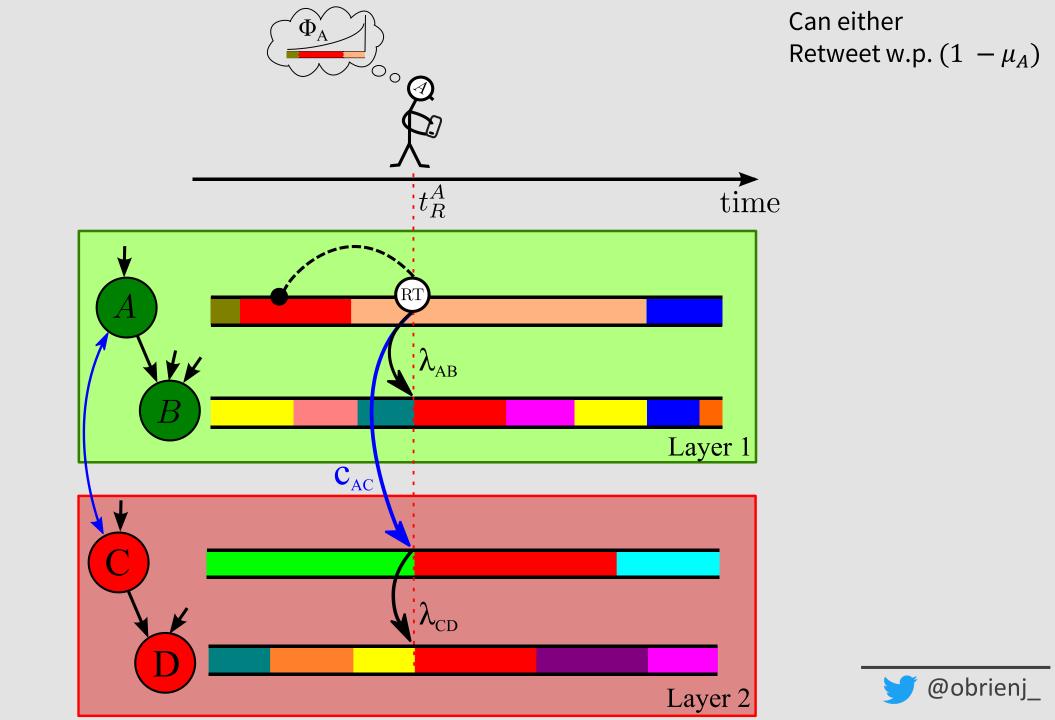
🈏 @obrienj_

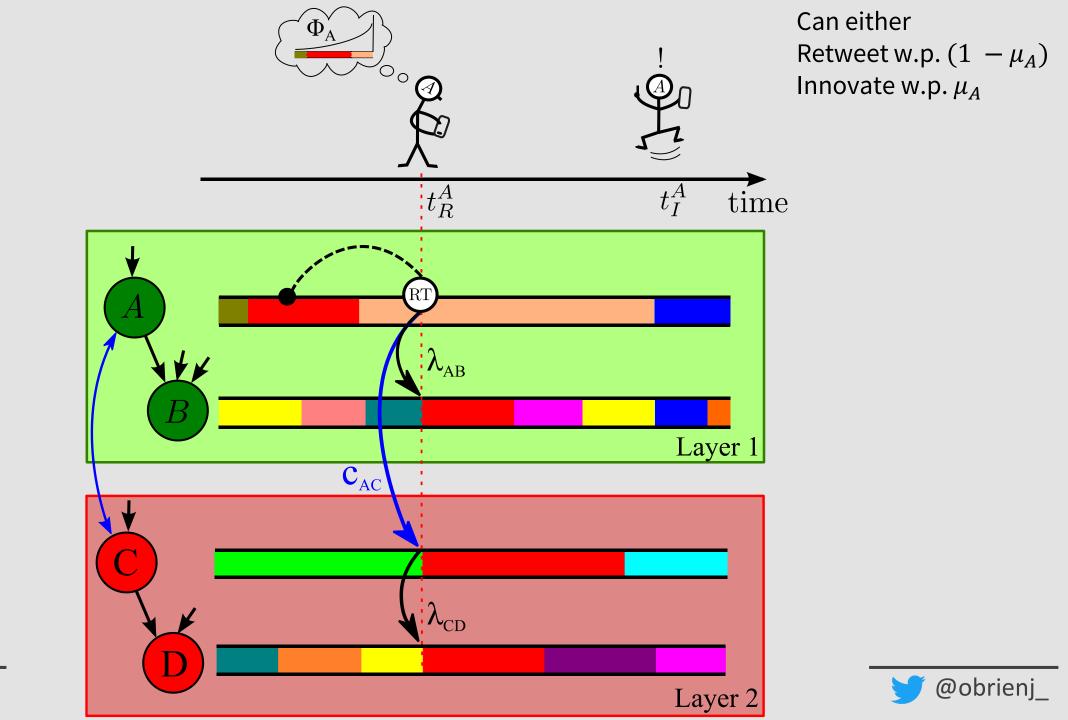


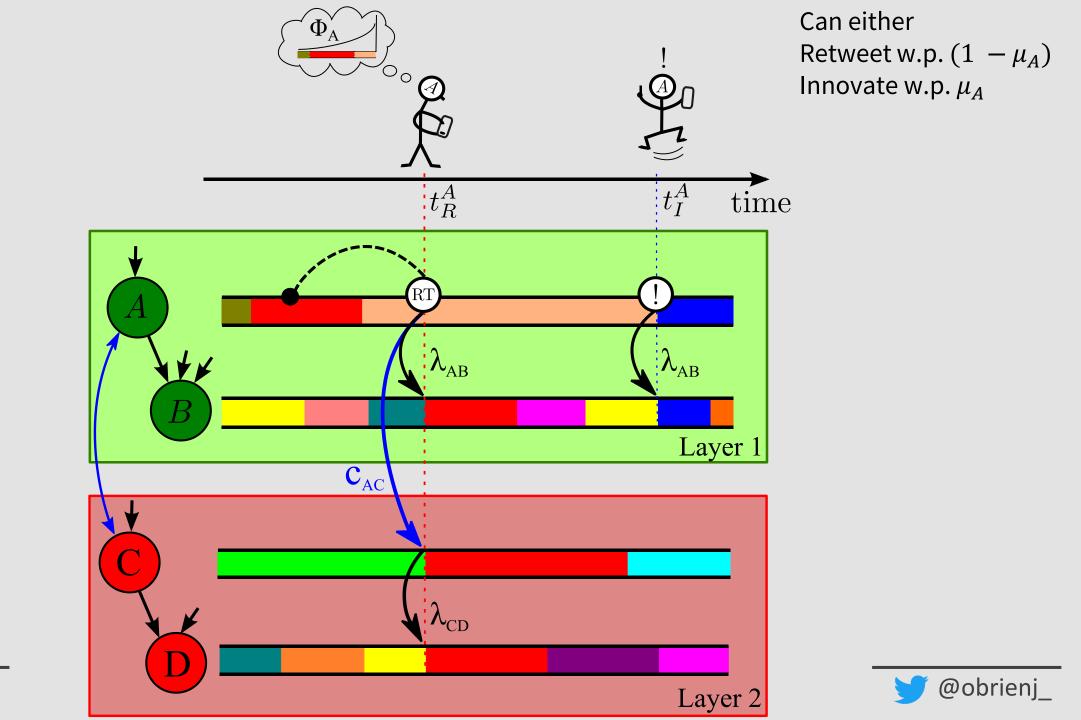
🈏 @obrienj_

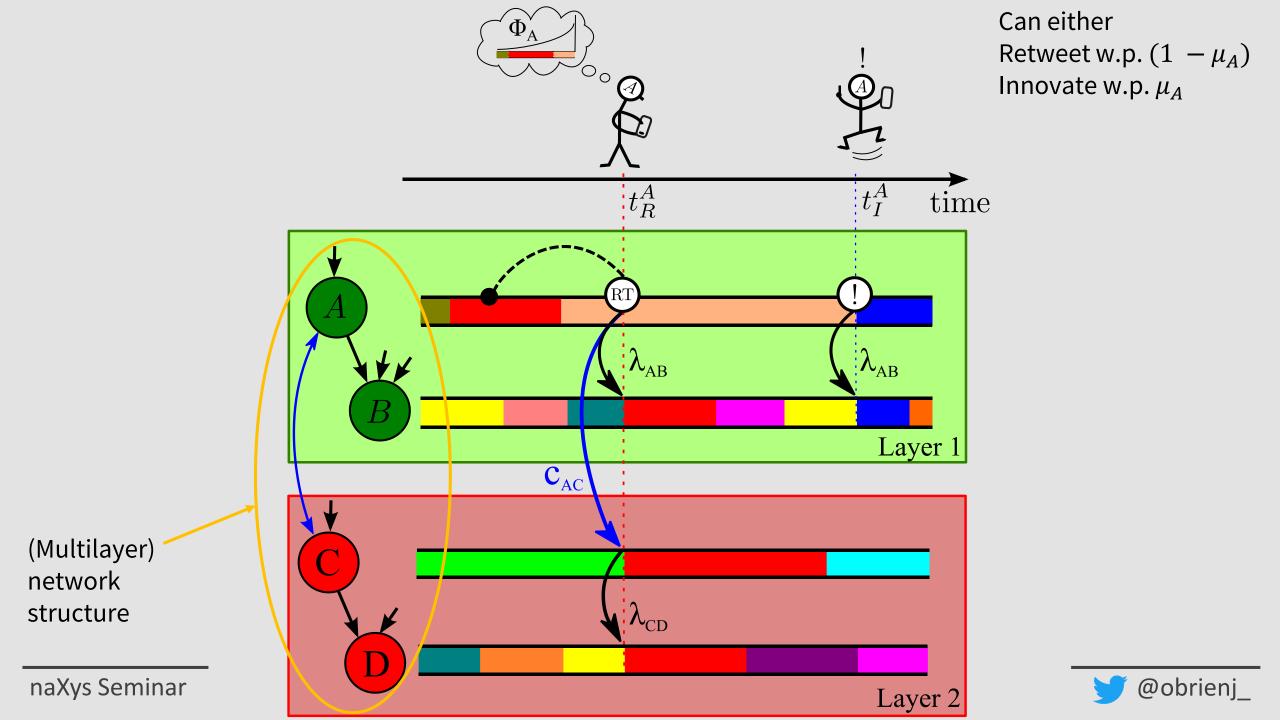


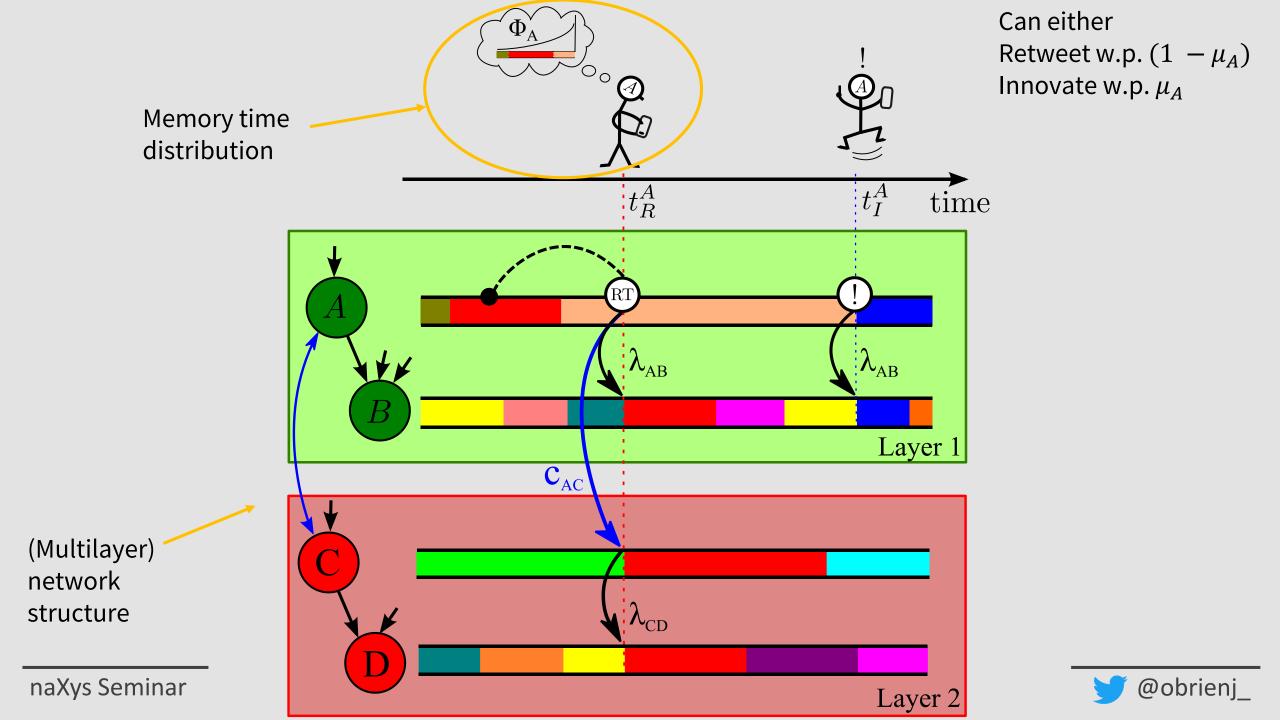


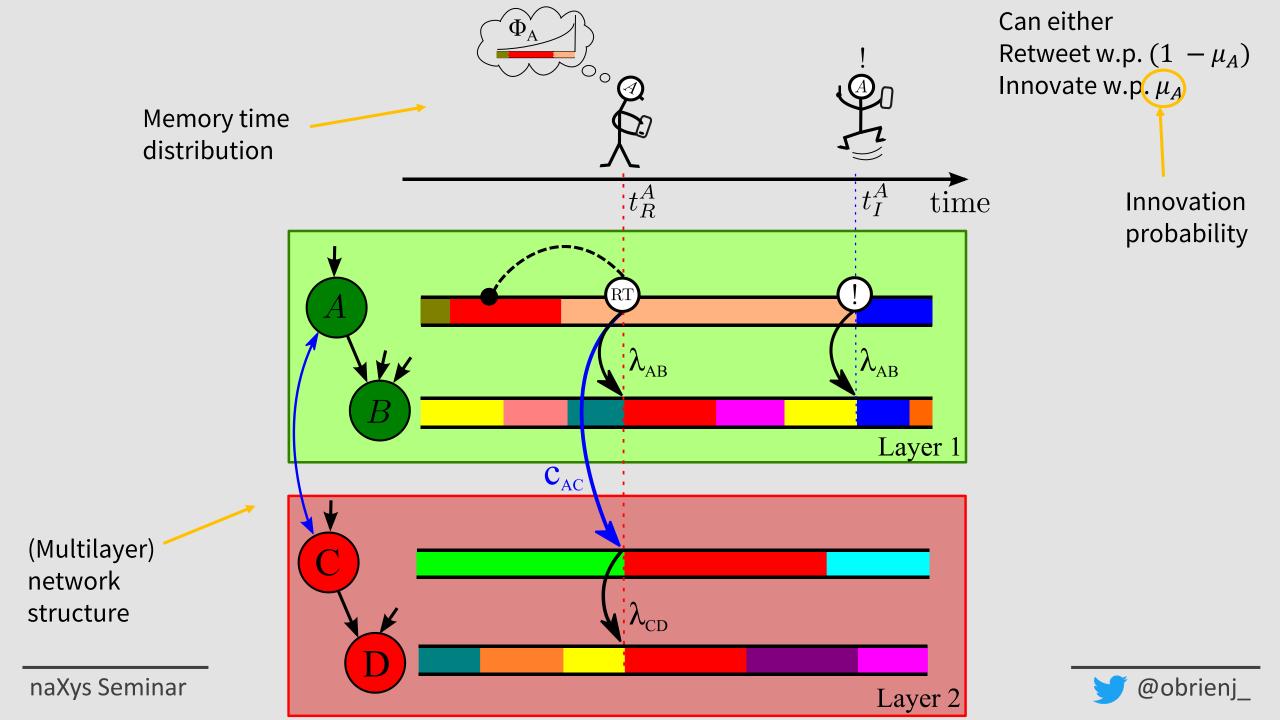


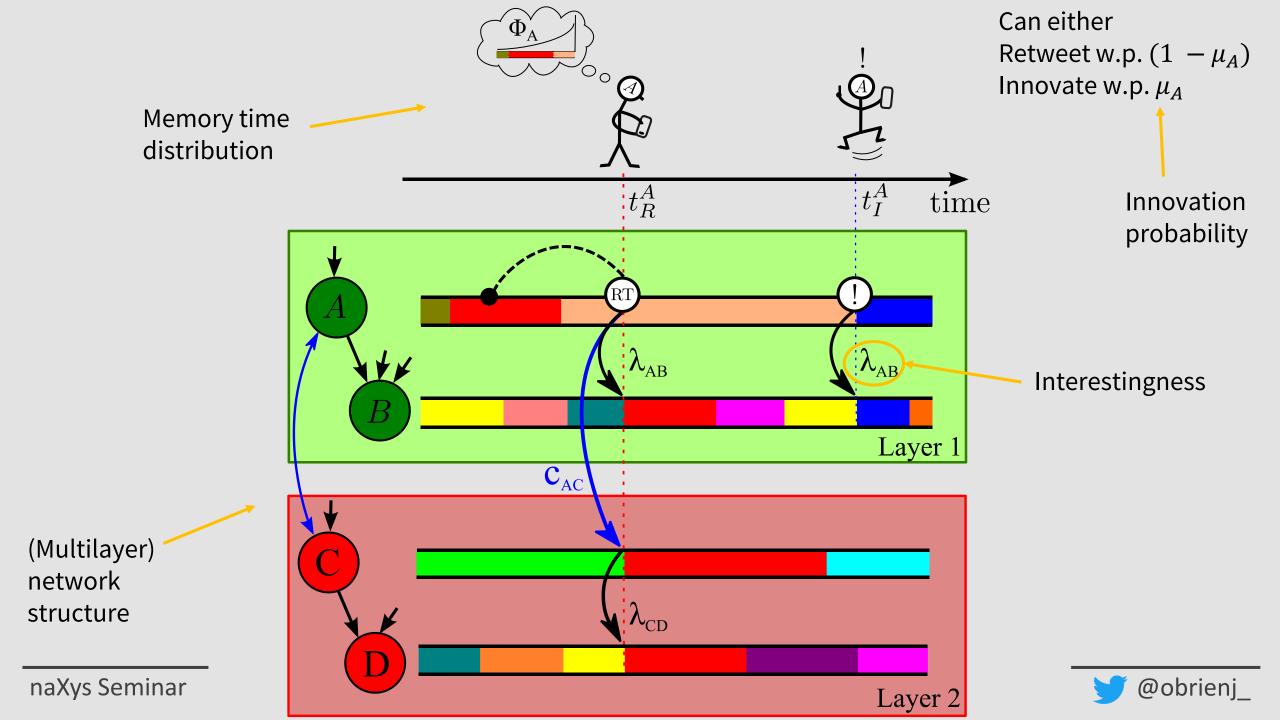


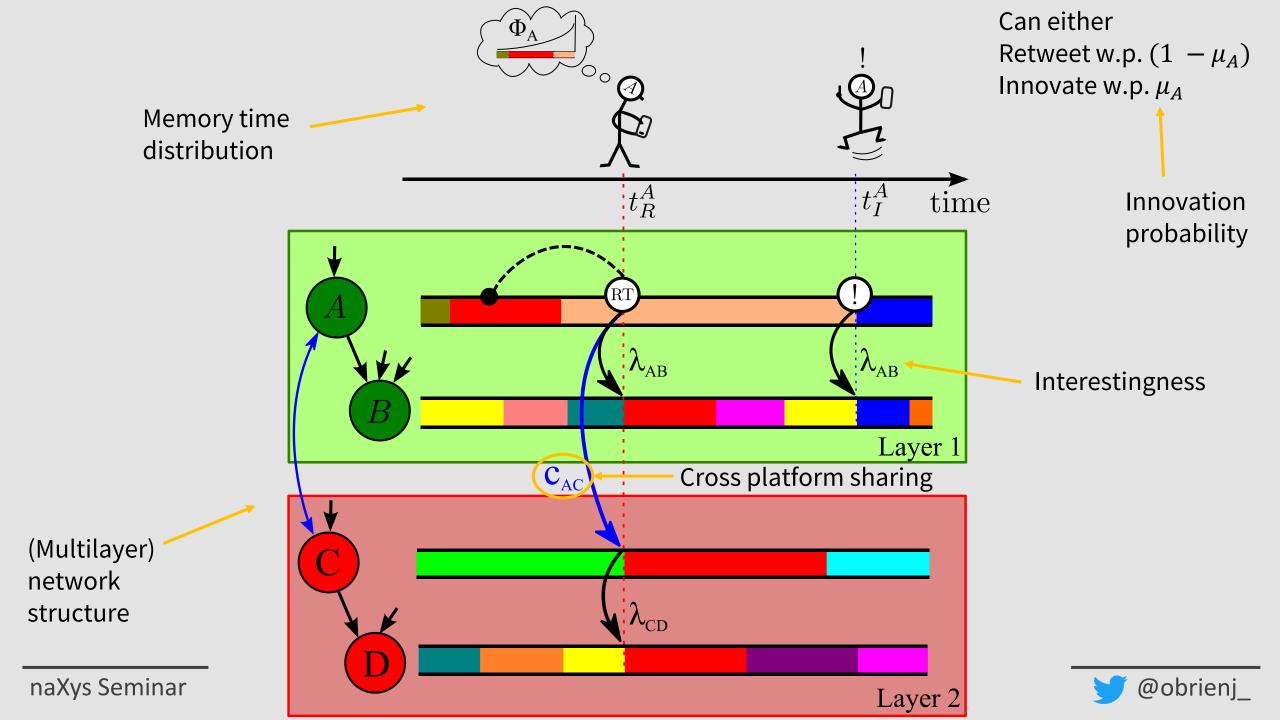


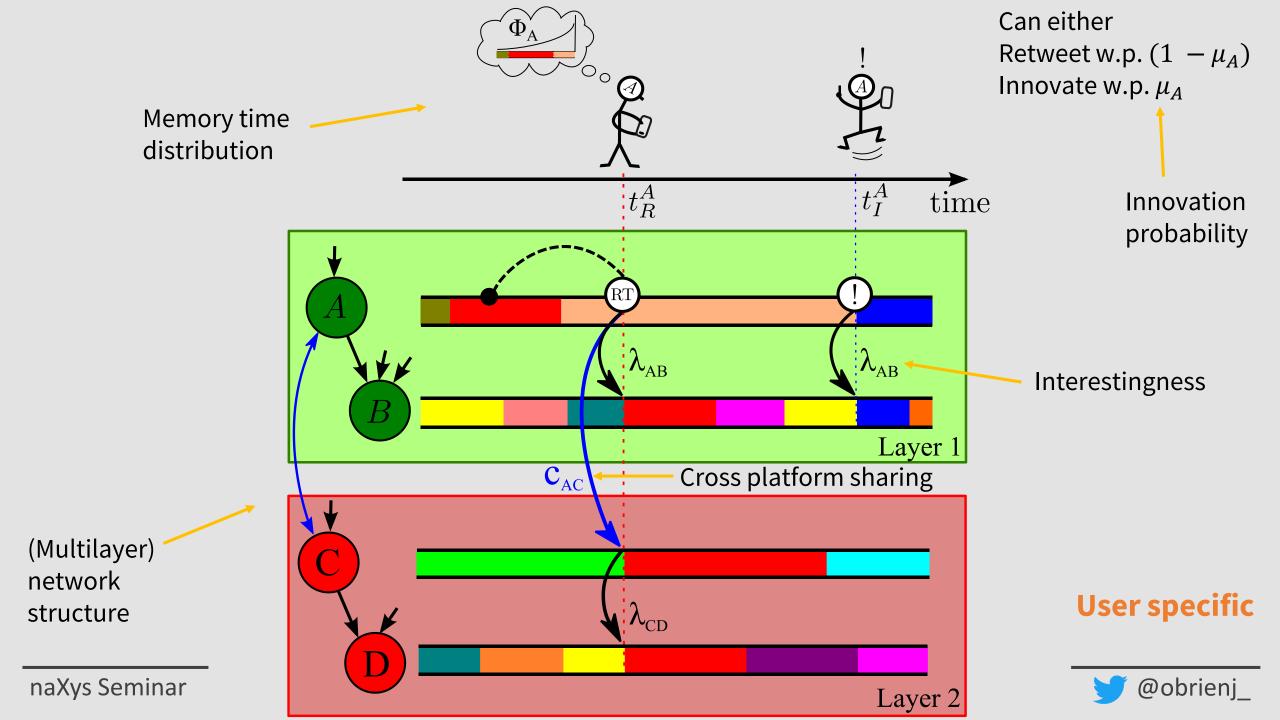










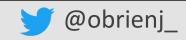


Maths behind the memes

Each edge, so either between 'friends' or the user's other accounts will have its own probability generating function which determines meme popularity

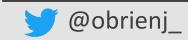
$$G_{ij}(a;x) = \int_{0}^{\infty} r_{i} \exp\left(-r_{i}\ell\right) \exp\left\{-(1-\mu_{i})\beta_{i} \int_{0}^{\min(\ell,a)} \mathrm{d}\tilde{r} \int_{0}^{a-\tilde{r}} \Phi_{i}(a-\tilde{r}-\tilde{t}) \left[1-R_{ij}(\tilde{t};x)\right] \, \mathrm{d}\tilde{t}\right\} \, \mathrm{d}\ell$$
network
structure
Interestingness
$$R_{ij}(a;x) = x \left(1-\lambda_{ij}+\lambda_{ij} \prod_{k} G_{jk}(a;x)\right) \left\{1-c_{ij}+c_{ij} \left[\prod_{l} \left(1-\lambda_{jl}+\lambda_{jl} \prod_{m} G_{lm}(a;x)\right)\right]\right\}.$$

Difficult to work with, but can do some analysis...



The so-called 'Mean Matrix'

$$m_{ij} \sim \frac{(c_{ij} + \lambda_{ij})(1 - \mu_i)\beta_i}{\mu_i \beta_i + \sum_k \lambda_{ki} \beta_k + \sum_k c_{ki} \beta_k + \sum_k [\lambda_{ki} \left(\sum_l c_{lk} \beta_l\right)]}$$



The so-called 'Mean Matrix'

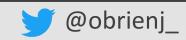
$$m_{ij} \sim \frac{(c_{ij} + \lambda_{ij})(1 - \mu_i)\beta_i}{\mu_i \beta_i + \sum_k \lambda_{ki} \beta_k + \sum_k c_{ki} \beta_k + \sum_k [\lambda_{ki} \left(\sum_l c_{lk} \beta_l\right)]}$$

Criticality of the system is determined by the largest eigenvalue of this matrix

$$\rho < 1 \implies \text{Subcritical}$$

 $\rho = 1 \implies \text{Critical}$

 $\rho > 1 \implies \text{Supercritical}$

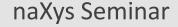


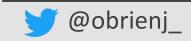
Monoplex with $\mu_i = 0$

In this case the mean matrix simply has elements given by

$$m_{ij} = \frac{\lambda_{ij}\beta_i}{\sum_k \lambda_{ki}\beta_k}$$

We can show that the largest eigenvalue in this case is exactly 1, a critical system, generalizing the result of Gleeson et al. (*PRX 2016*).





Monoplex with $\mu_i \geq 0$

In this case the mean matrix changes slightly

$$m_{ij} = \frac{(1 - \mu_i)\lambda_{ij}\beta_i}{\mu_i\beta_i + \sum_k \lambda_{ki}\beta_k}$$

Using perturbative arguments we can show that in this case the largest eigenvalue is less than 1, i.e., a subcritical system.

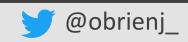


Multiplex with $c_{ij}, \mu_i \geq 0$

We are now back in the case we showed originally

$$m_{ij} \sim \frac{(c_{ij} + \lambda_{ij})(1 - \mu_i)\beta_i}{\mu_i\beta_i + \sum_k \lambda_{ki}\beta_k + \sum_k c_{ki}\beta_k + \sum_k [\lambda_{ki}\left(\sum_l c_{lk}\beta_l\right)]}$$

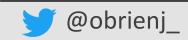
In this case the analysis is difficult but some perturbative results may still be obtained, and in fact for small crossover probabilities we find that the change depends on the existence (or not) of a dominant layer...



Multiplex with $c_{ij}, \mu_i \geq 0$

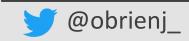
If a dominant layer exists, the criticality of the system is dependent purely on this dominant layer. $\rho_1 > \rho_2$





Multiplex with $c_{ij}, \mu_i \geq 0$

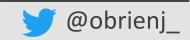
If a dominant layer exists, the criticality of the system is dependent purely on this dominant layer. $ho_1 >
ho_2$



Multiplex with $\ c_{ij}, \mu_i \geq 0$

If a dominant layer exists, the criticality of the system is dependent purely on this dominant layer. $ho_1 >
ho_2$

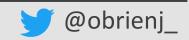
If two or more layer's largest eigenvalue are approximately equal however the result is less clear. $ho_1 pprox
ho_2$



Multiplex with $\ c_{ij}, \mu_i \geq 0$

If a dominant layer exists, the criticality of the system is dependent purely on this dominant layer. $ho_1 >
ho_2$

If two or more layer's largest eigenvalue are approximately equal however the result is less clear. $ho_1 pprox
ho_2$

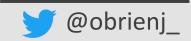


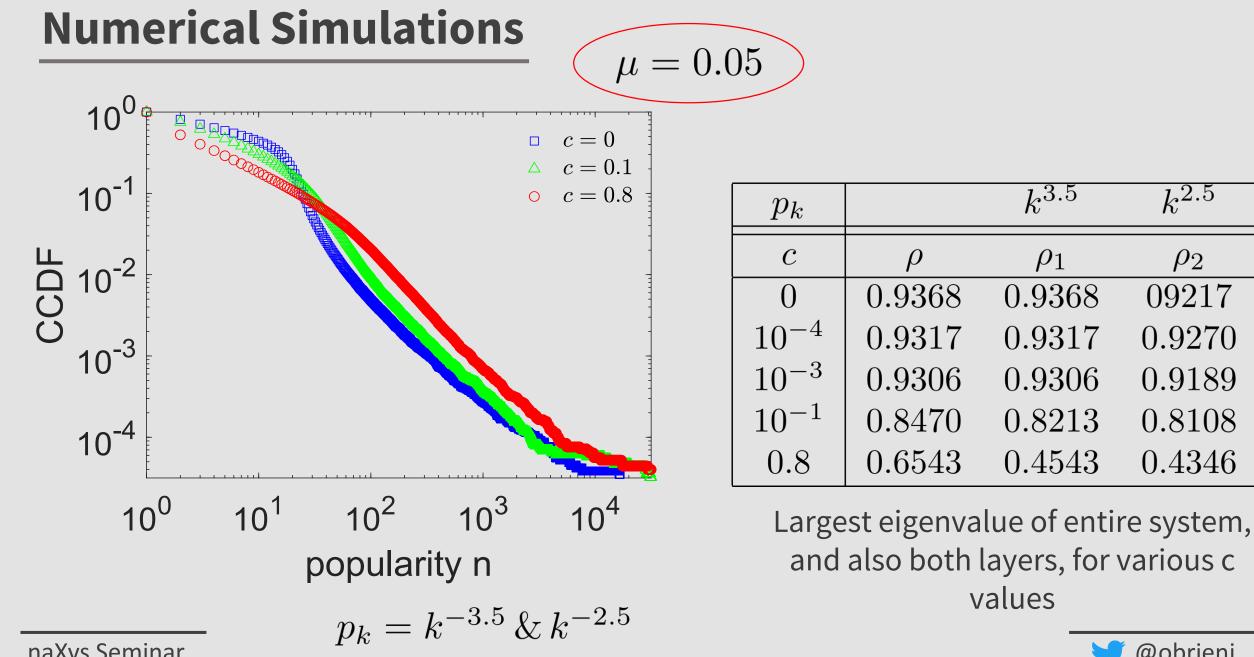
Multiplex with $\ c_{ij}, \mu_i \geq 0$

If a dominant layer exists, the criticality of the system is dependent purely on this dominant layer. $\rho_1 > \rho_2$

If two or more layer's largest eigenvalue are approximately equal however the result is less clear. $ho_1 pprox
ho_2$

and as such we resort to numerical simulations.



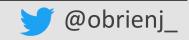


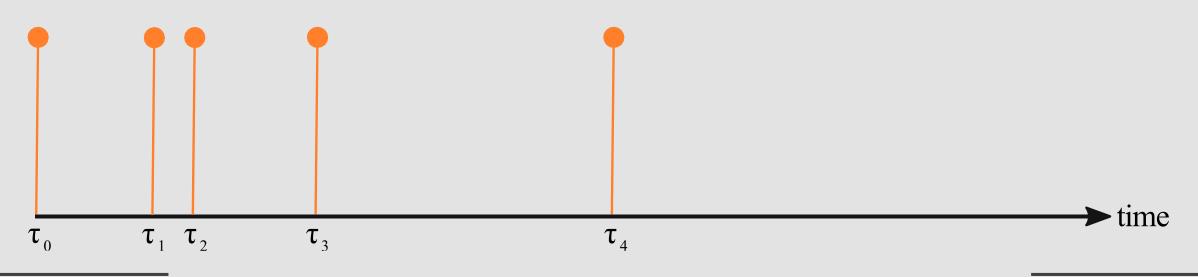
naXys Seminar

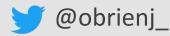
@obrienj_

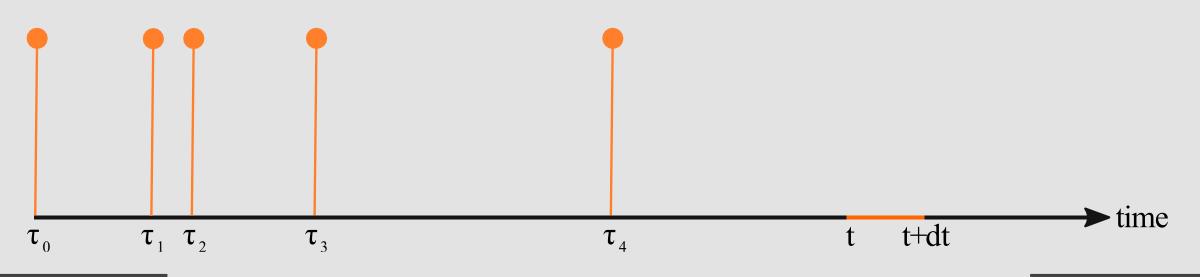
Predicting Content Popularity and Self-exciting Processes

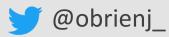
- J. Cheng, L. Adamic, P. A. Dow, J. M. Kleinberg, and J. Leskovec. Can cascades be predicted? *Proc. WWW23*, 925–936. ACM, 2014.
- J. M. Miotto and E. G. Altmann. Predictability of extreme events in social media. *PLoS One*, 9(11):e111506, 2014.
- L. Weng, F. Menczer, and Y. Ahn. Virality prediction and community structure in social networks. *Scientific Reports*, 3:2522, 2013.
- R. Bandari, S. Asur, and B. A. Huberman. The pulse of news in social media: Forecasting popularity. *Proc. AAAI CWSM6*, 2012.
- ...
- S. Mishra, M. A. Rizoiu, and L. Xie. Feature driven and point process approaches for popularity prediction. *Proc. ACM IIKM*, 1069–1078. ACM, 2016.
- Q. Zhao, M. A. Erdogdu, H. Y. He, A. Rajaraman, and J. Leskovec. SEISMIC: A self-exciting point process model for predicting tweet popularity. *Proc. ACM SIGKDD*, 1513–1522. ACM, 2015.
- R. Kobayashi and R. Lambiotte. TiDeH: Time-dependent Hawkes process for predicting retweet dynamics. *Proc. AAAI CWSM10*, 2016.
- M. A. Rizoiu, L. Xie, S. Sanner, M. Cebrian, H. Yu, and P. Van Hentenryck. Expecting to be HIP: Hawkes intensity processes for social media popularity. *Proc. WWW26*, 735–744. ACM, 2017.
- ...
- Work generally based on intensive numerical computation rather than mathematically tractable models.



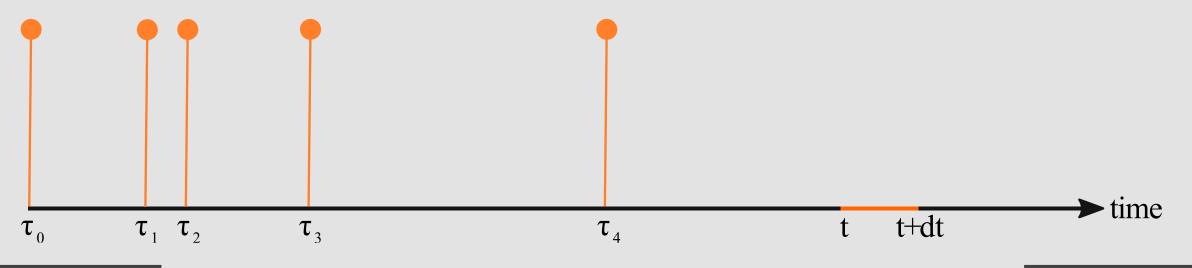


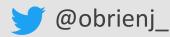




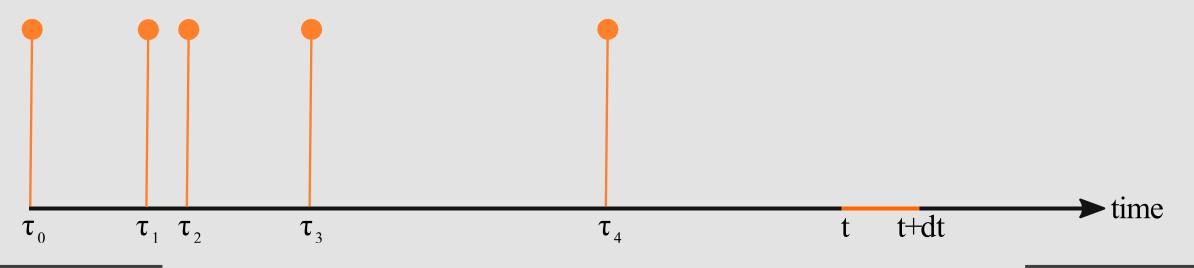


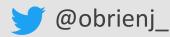
Process	Expected no. of events
Poisson process	λ dt
Inhomogeneous Poisson process	$\lambda(t) dt$
Hawkes process	$\left[\mu(t) + \Sigma_i \xi \phi \left(t - \tau_i\right)\right] dt$



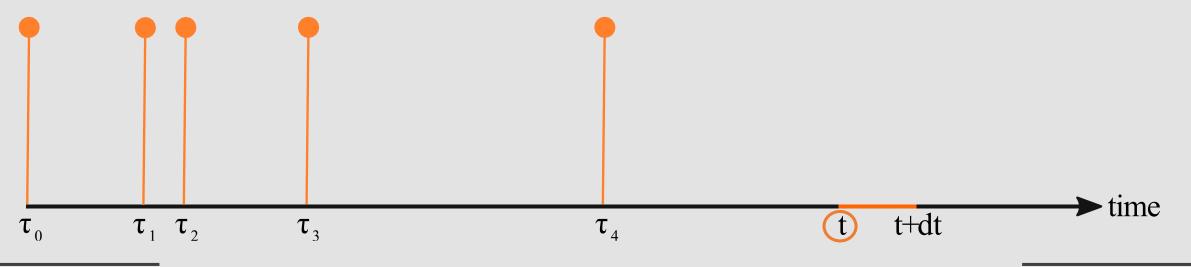


Process	Expected no. of events
Poisson process	λdt
Inhomogeneous Poisson process	$\lambda(t) dt$
Hawkes process	$[\mu(t) + \Sigma_i \xi \phi (t - \tau_i)] dt$



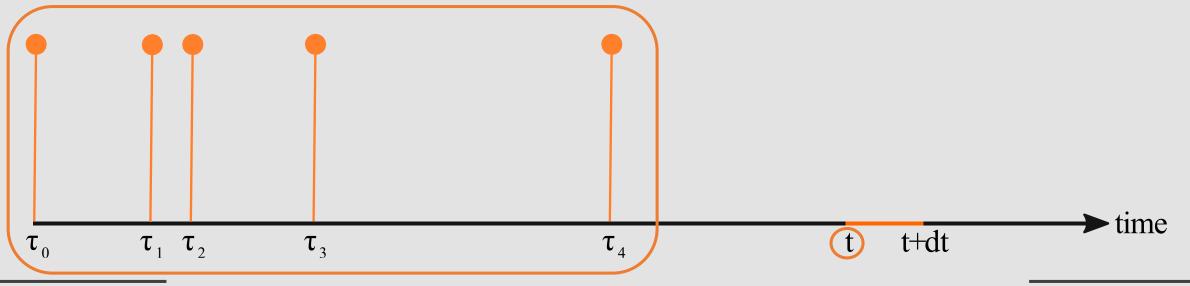


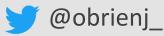
Process	Expected no. of events
Poisson process	λ dt
Inhomogeneous Poisson process	$\lambda(t) dt$
Hawkes process	$[\mu(t) + \Sigma_i \xi \phi (t - \tau_i)] dt$

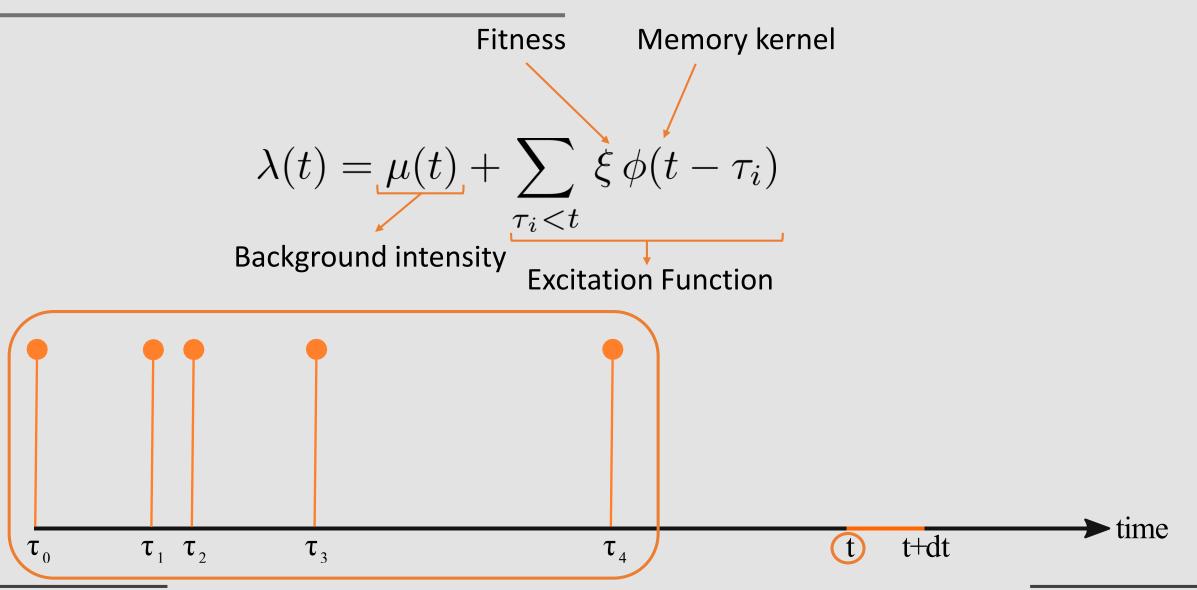


🈏 @obrienj_

Process	Expected no. of events
Poisson process	λdt
Inhomogeneous Poisson process	$\lambda(t) dt$
Hawkes process	$\left[\mu(t) + \Sigma_i \xi \phi \left(t - \tau_i\right)\right] dt$







@obrienj_

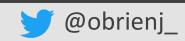
Example Kernels

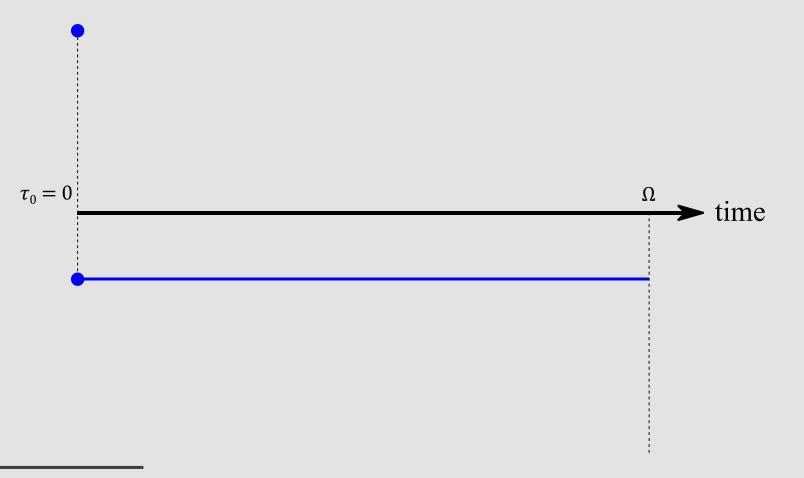
The most well known kernel is constant background intensity with exponential memory, particularly in finance.

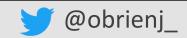
$$\lambda(t) = \lambda_0 + \xi \sum_{\tau_i < t} \beta e^{-\beta(t-\tau_i)},$$

another commonly used kernel is that of the shifted power-law generally used in seismology.

$$\lambda(t) = \lambda_0 + \xi \sum_{\tau_i < t} \beta \, c^\beta \, (t - \tau_i + c)^{-(1+\beta)}.$$





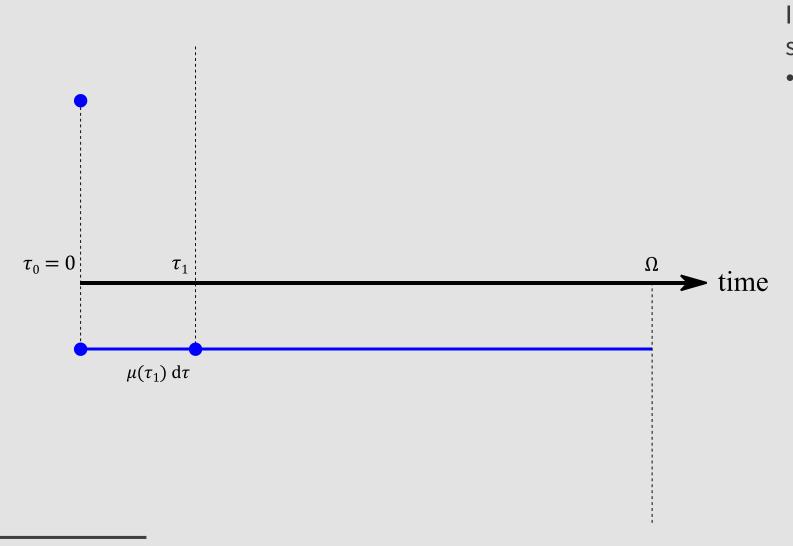


 $au_0 = 0$ Ω time

In each time interval (t, t + dt)someone may

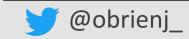
• Reply to the original post w.p. $\mu(t) dt$

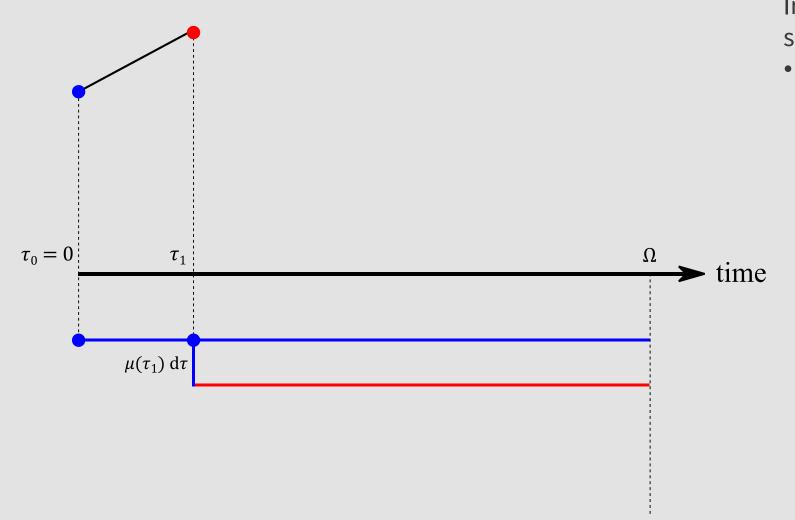




In each time interval (t, t + dt)someone may

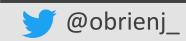
• Reply to the original post w.p. $\mu(t) dt$

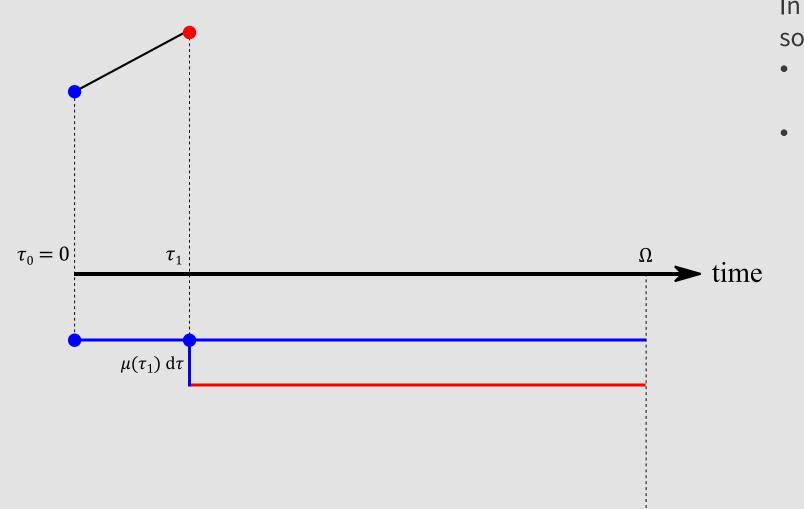




In each time interval (t, t + dt)someone may

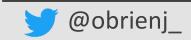
• Reply to the original post w.p. $\mu(t) dt$

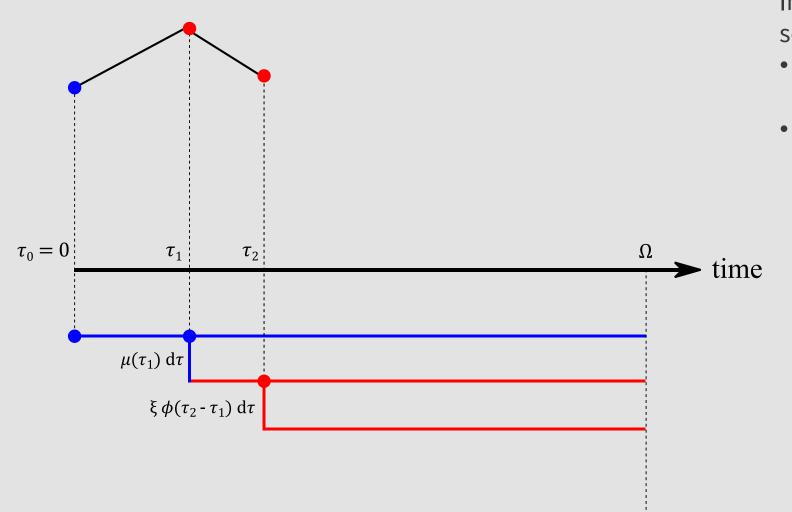




In each time interval (t, t + dt)someone may

- Reply to the original post w.p. $\mu(t) dt$
- Reply to a reply made at time τ_i w.p. $\xi \phi(t - \tau_i) dt$

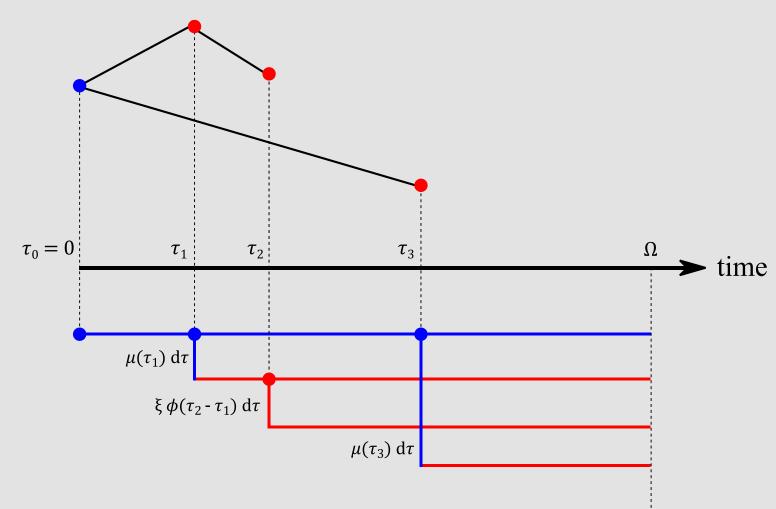




In each time interval (t, t + dt)someone may

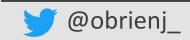
- Reply to the original post w.p. $\mu(t) dt$
- Reply to a reply made at time τ_i w.p. $\xi \phi(t - \tau_i) dt$

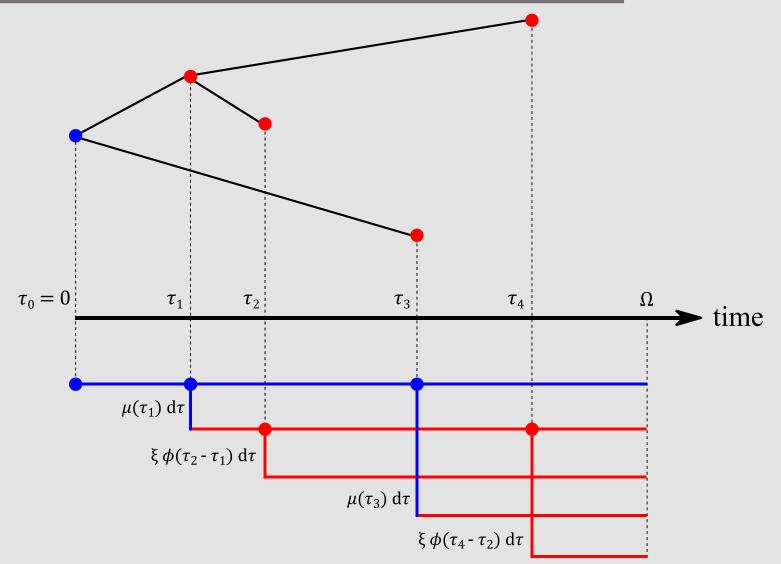




In each time interval (t, t + dt)someone may

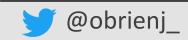
- Reply to the original post w.p. $\mu(t) dt$
- Reply to a reply made at time τ_i w.p. $\xi \phi(t - \tau_i) dt$

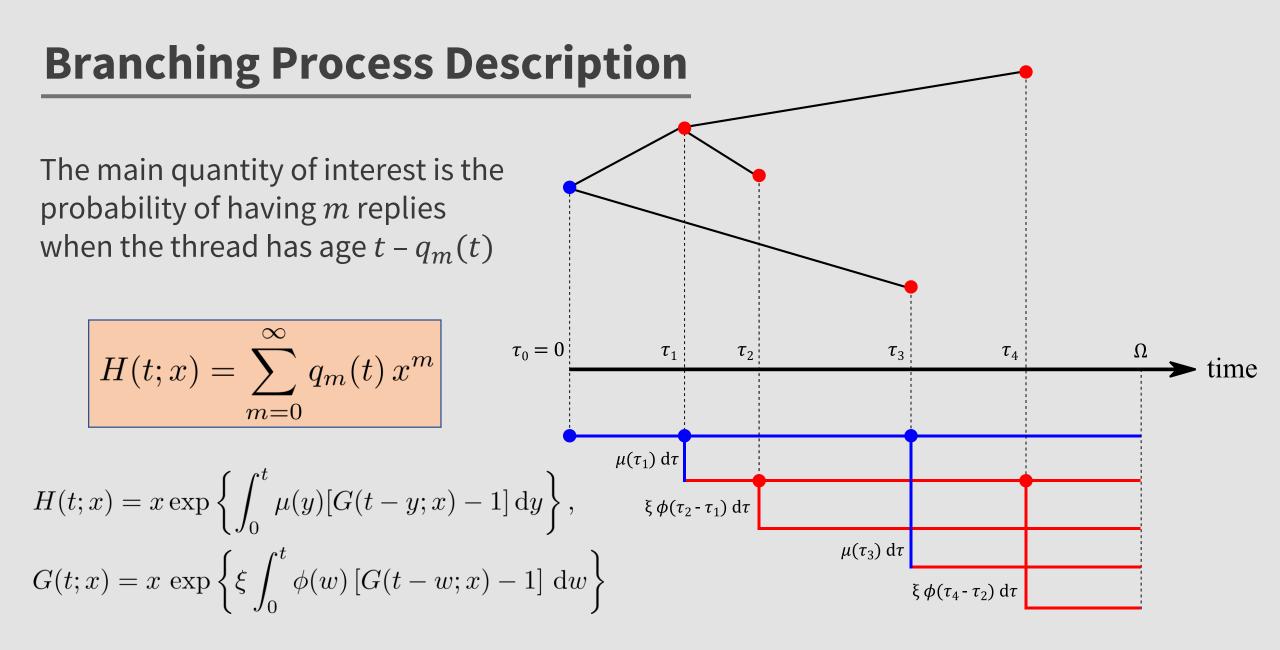


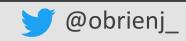


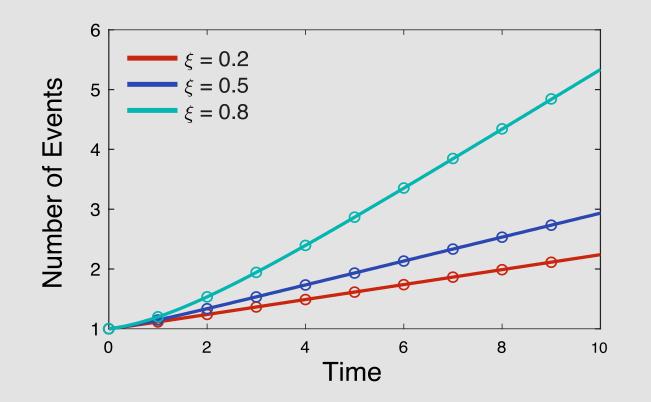
In each time interval (t, t + dt)someone may

- Reply to the original post w.p. $\mu(t) dt$
- Reply to a reply made at time τ_i w.p. $\xi \phi(t - \tau_i) dt$









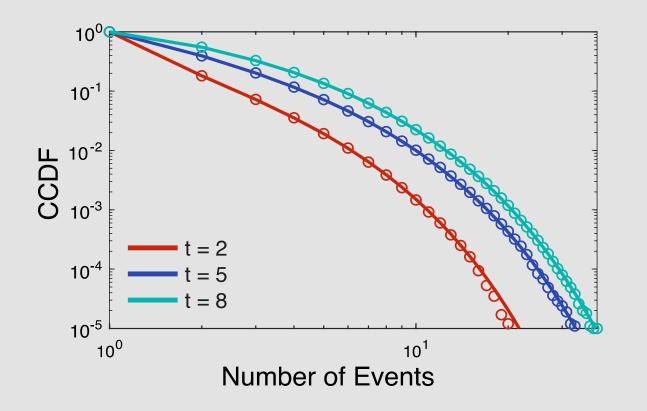
Calculate the expected number of events in Laplace space through

$$\hat{m}(s) = \frac{1}{s} + \frac{\hat{\mu}(s)}{s \left[1 - \xi \,\hat{\phi}(s)\right]},$$

which may be analytically analysed in some scenarios e.g., exponential case

$$m(t) = 1 + \frac{\lambda_0}{1 - \xi} \left[t + \frac{\xi}{\beta(1 - \xi)} \left(e^{-\beta(1 - \xi)t} - 1 \right) \right]$$



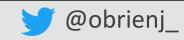


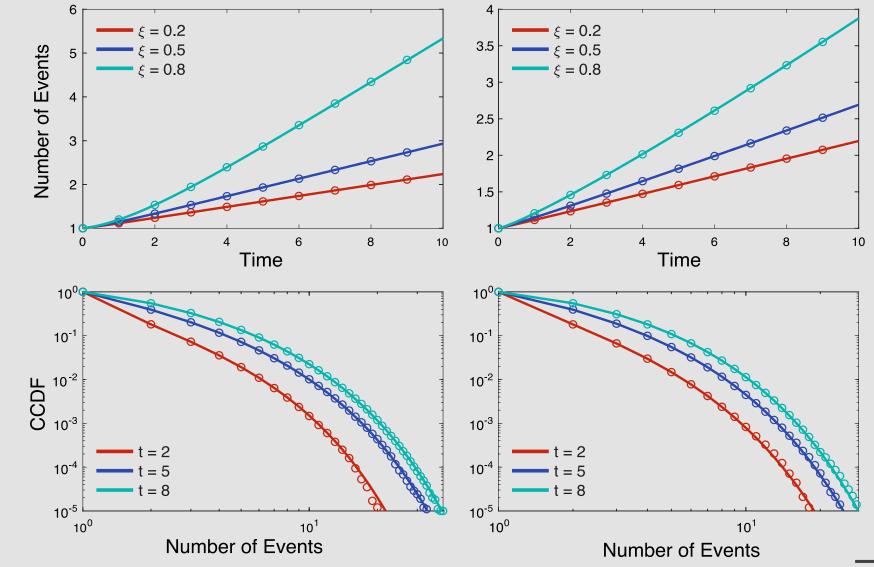
Determine the distribution at any time via

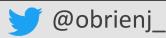
$$q_m(t) = \frac{1}{m!} \frac{\mathrm{d}^m}{\mathrm{d}x^m} H(t;x) \Big|_{x=0}$$
$$= \frac{1}{2\pi i} \oint_C H(t;x) x^{-(m+1)} \,\mathrm{d}x,$$

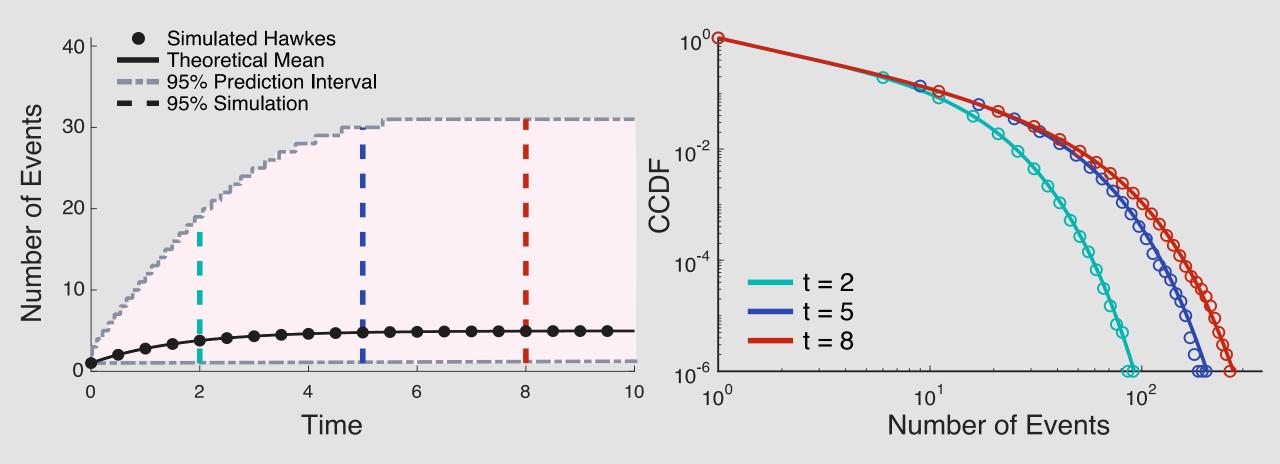
which is evaluated numerically through inverse fast Fourier transforms.

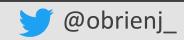




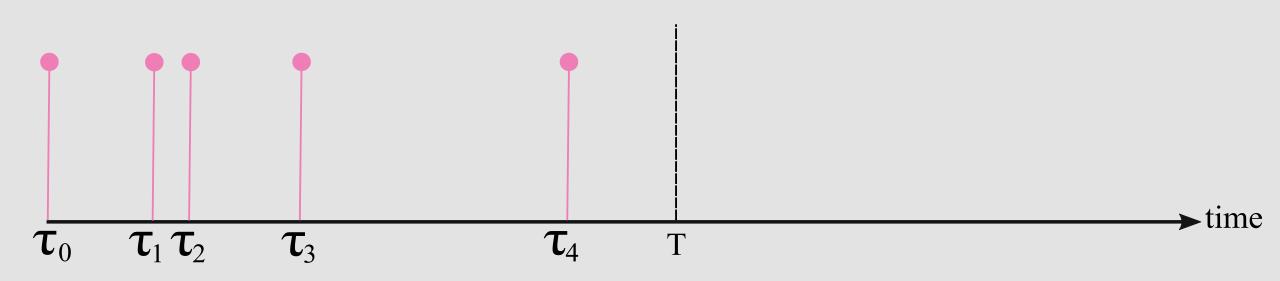


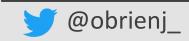


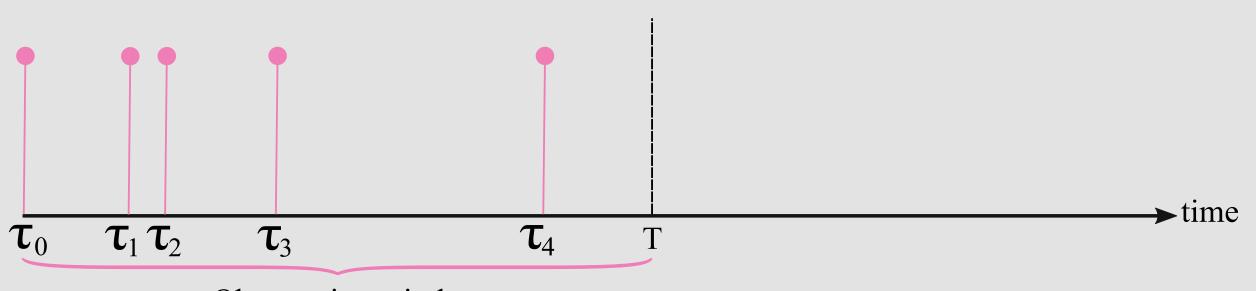




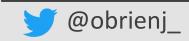
What About Prediction?

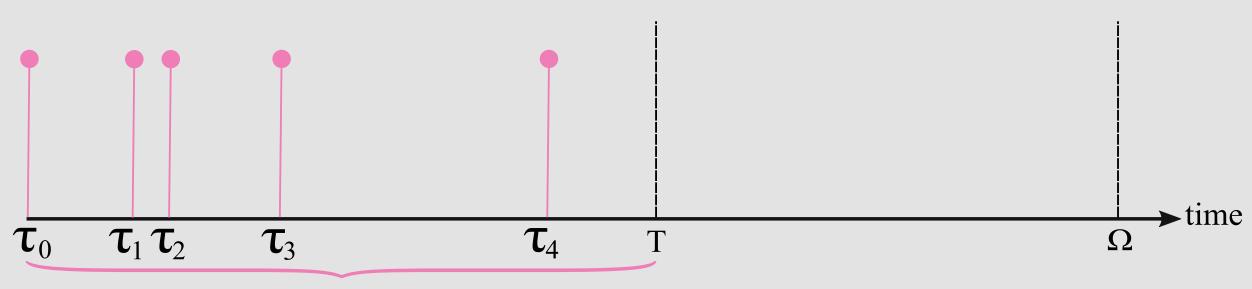






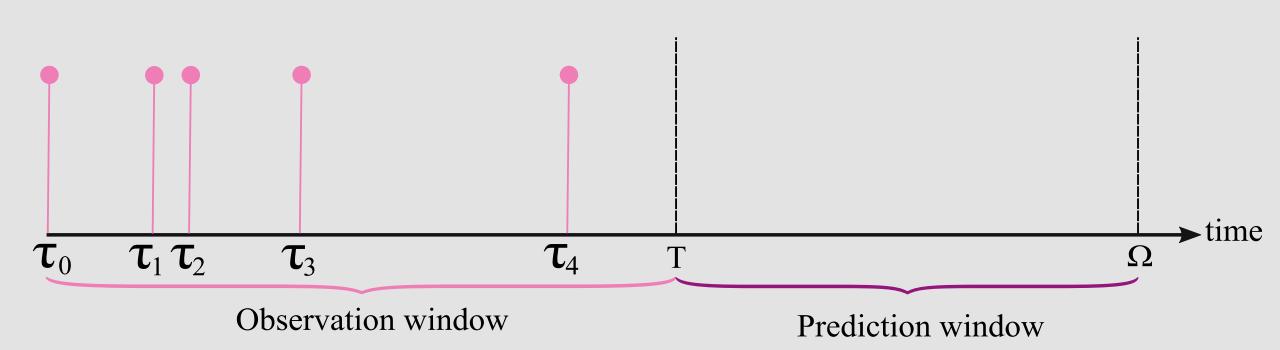
Observation window



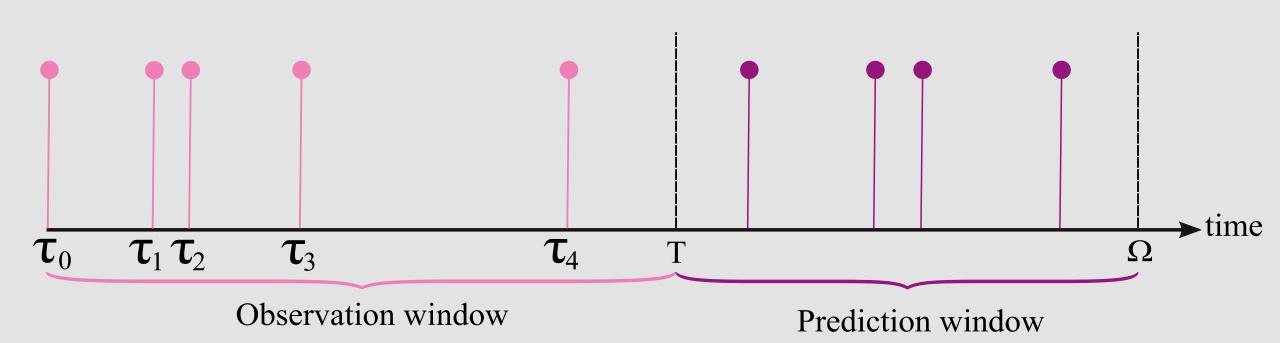


Observation window

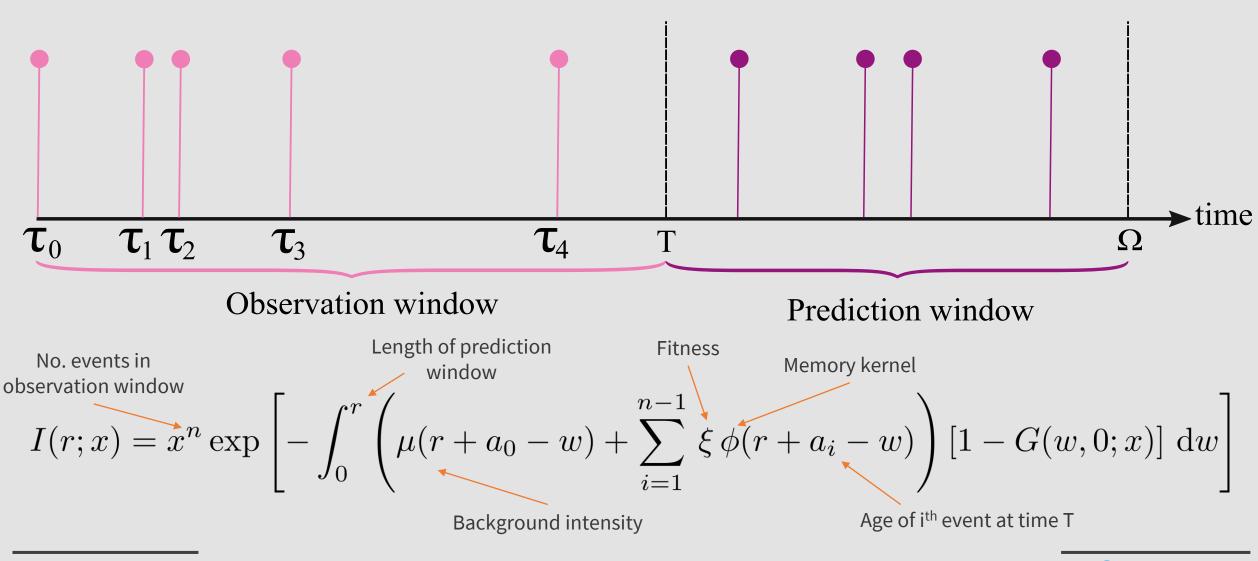


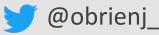


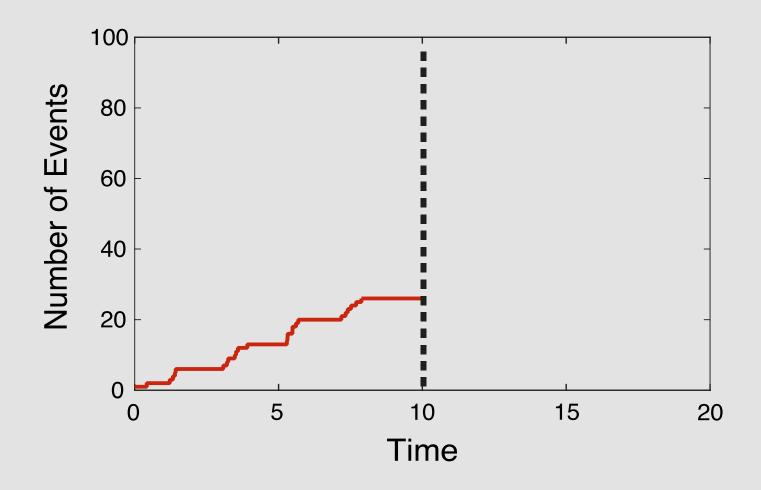


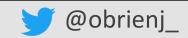


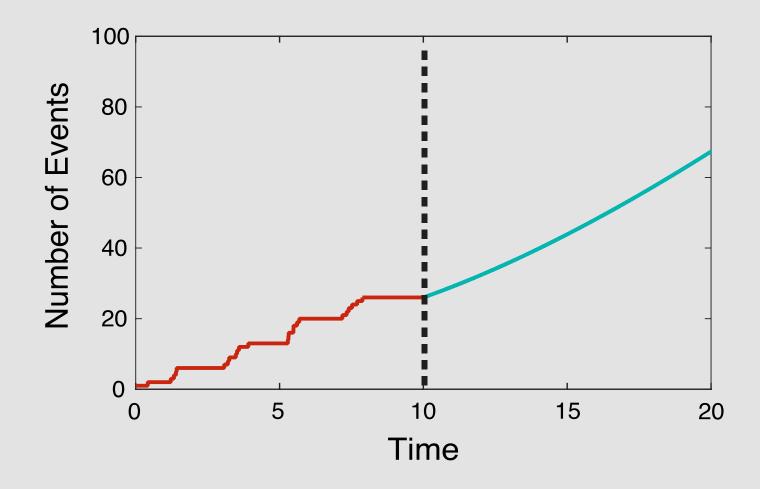


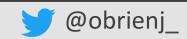


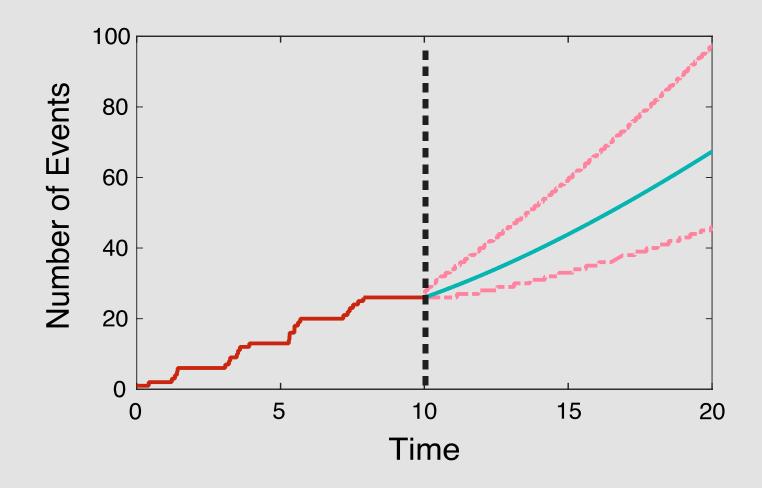


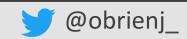


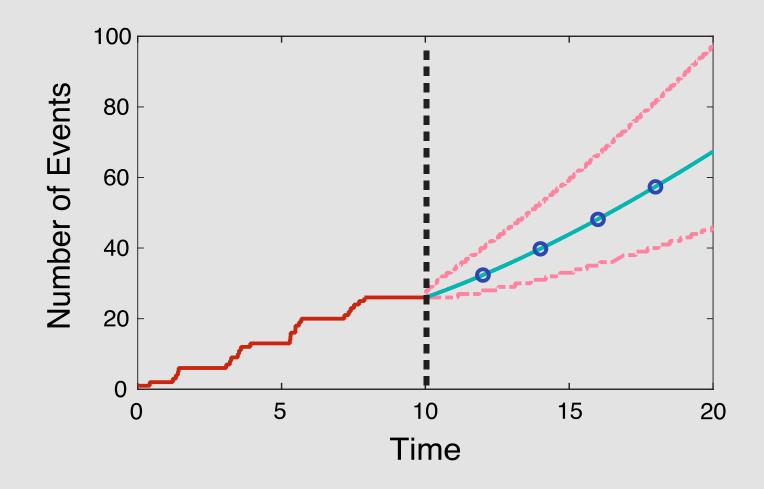


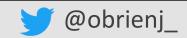


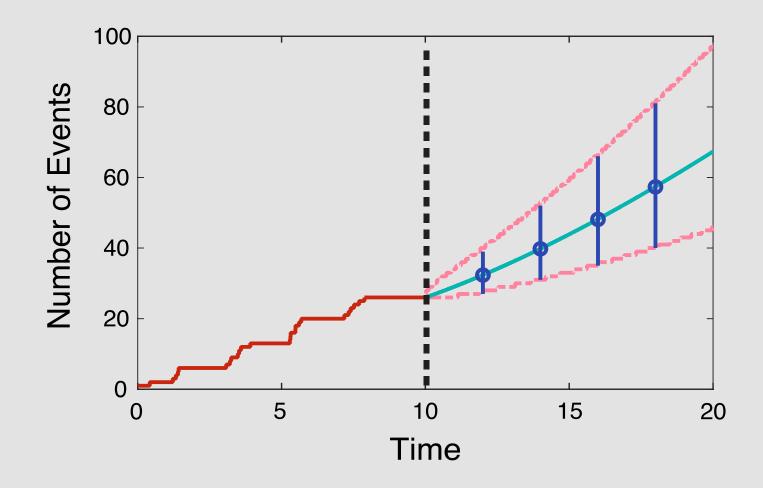


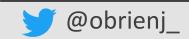


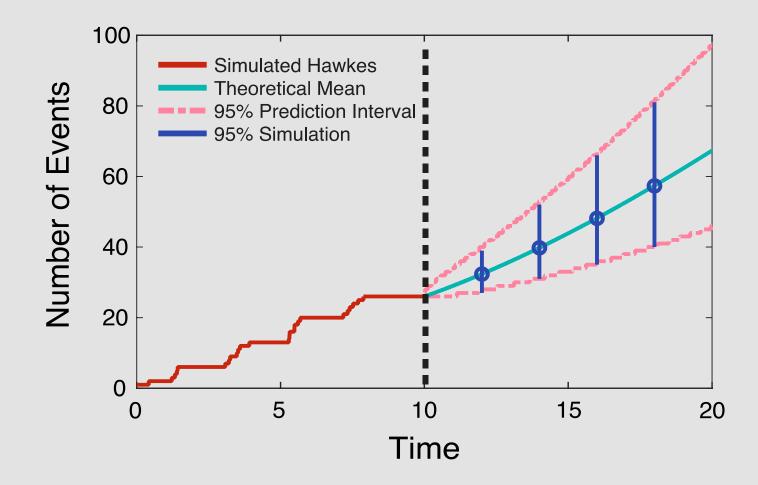


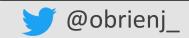




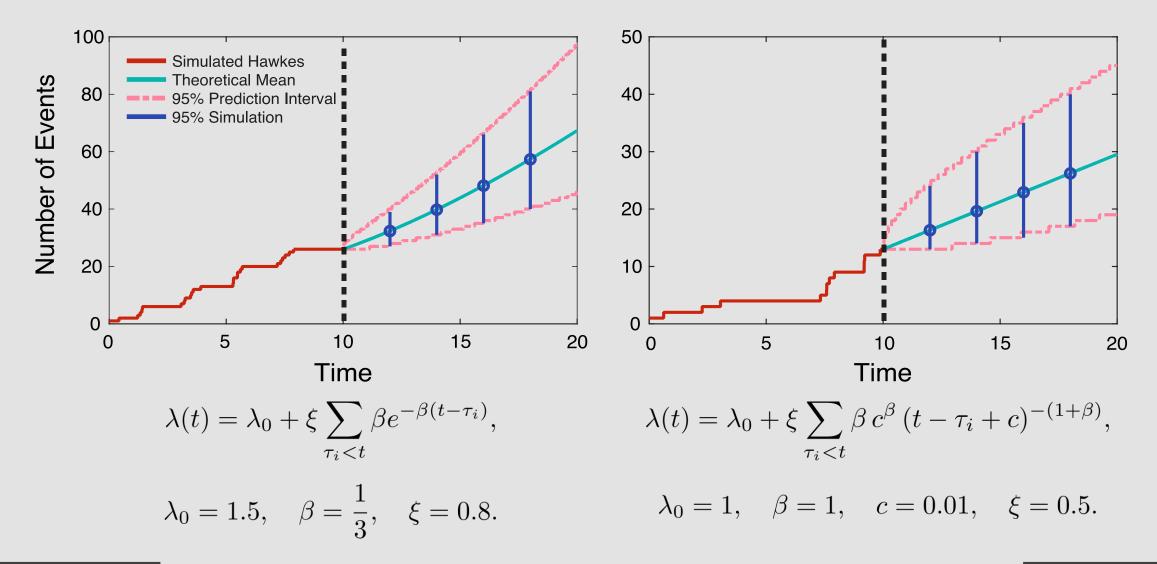


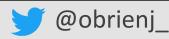






Simulation Results

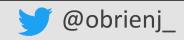


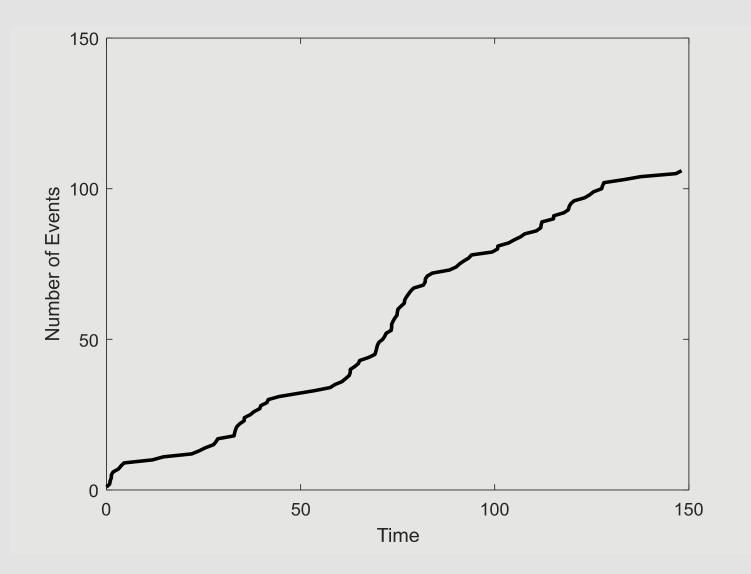


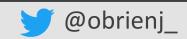
- Spanish language discussion board created in 2003 with the purpose of talking about cars.
- Now the 52nd most visited website in Spain with a wide-range of topics.
- Follows the usual dynamics of a discussion board where a user creates a thread which receives replies that themselves may be replied to. Threads are **ordered on the front page** shows at most 40 by time of last post.
- The longer the thread is on the front page more likely it is to be seen and replied to, so can be thought of as a **self-exciting process**.

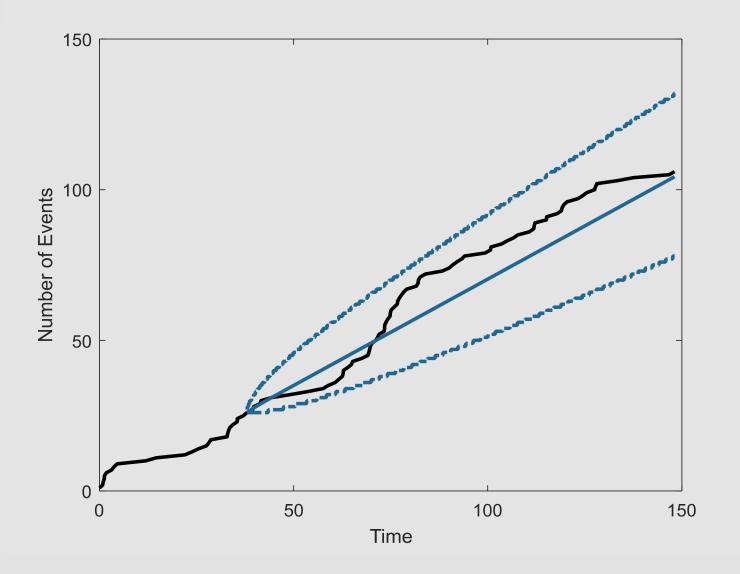
Un mallorquín recupera a su hija secuestrada durante 8 años por su madre B.Samaritano	Hoy 16:00 Hidra 🔰	
🗹 Me la quieren liar por Telegram. +PRV +CAPTURAS +POSIBLETROLEO ([] 1 2 3 4 5 6 7 8 Última Página) The Treeman	Hoy 16:00 TacoDeBillar 🔰	
Venga shures toca disfrutar ZKB	Hoy 16:00 DonPingPong 🔰	
Di Si o No y te vas. (1 1 2 3 4 5) Joe Capacho	Hoy 16:00 comilk 🔰	
∠A A o la B? ([] 1 2 3) Reydelaspajas	Hoy 16:00 Amster 🔰	
Qué le pasa a Federico con Greta??? (1 2) UnCojonudo	Hoy 16:00 S.Dev 🔰	
Section 2012 Secti	Hoy 16:00 MrWrong 🔰	
✓ PEÑA ESQUIZA DEPORTIVA Vol. 646 "LALETI, LA PUTITA DEL BARSA" -cholo +prv (□ 1 2 3 4 5 6 7 8 Última Página) Red John	Hoy 16:00 Pinfly22 🔰	1.814 13.701
Vendo grupo de WeChat de 2011 por 350€ Alacran	Hoy 16:00 Reiben 🔰	

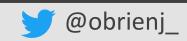
Pág 1 de 69 1 2 3 4 5 6 7 8 9 10 11 51 > Último » 🔻

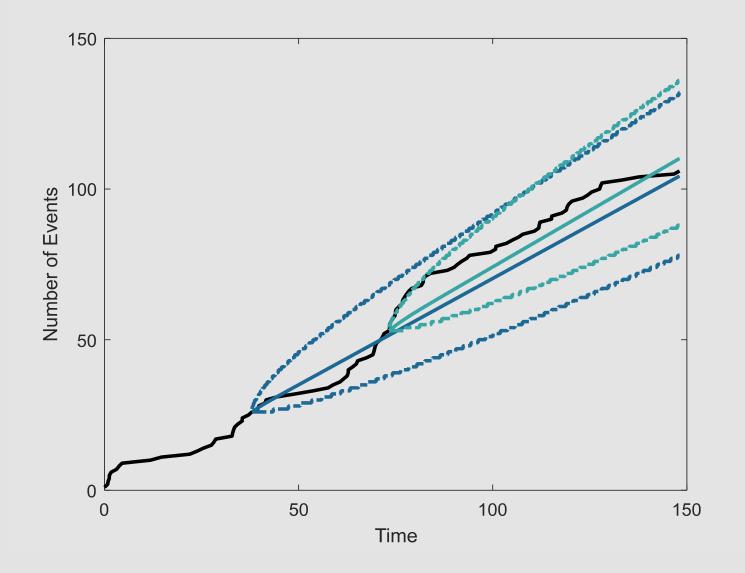


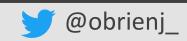


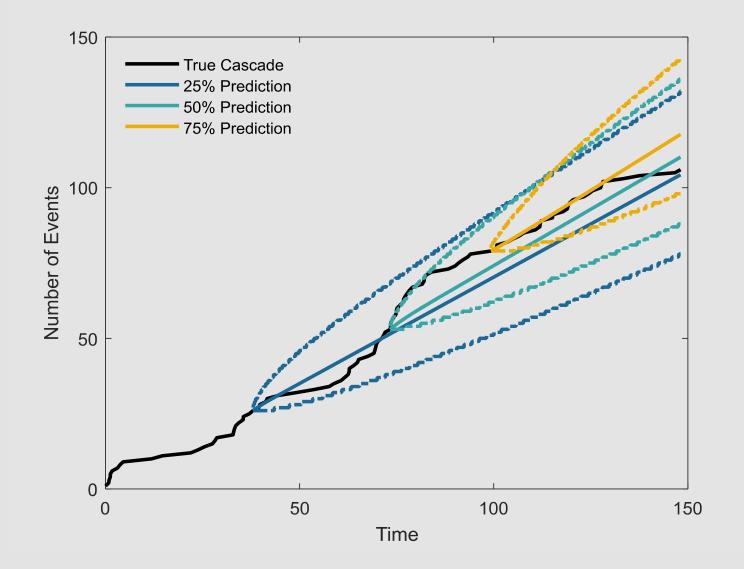


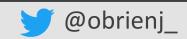


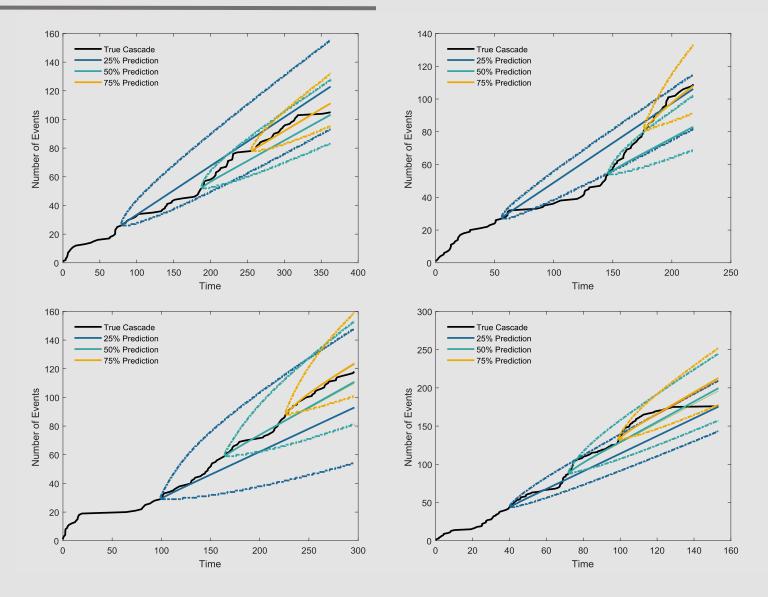








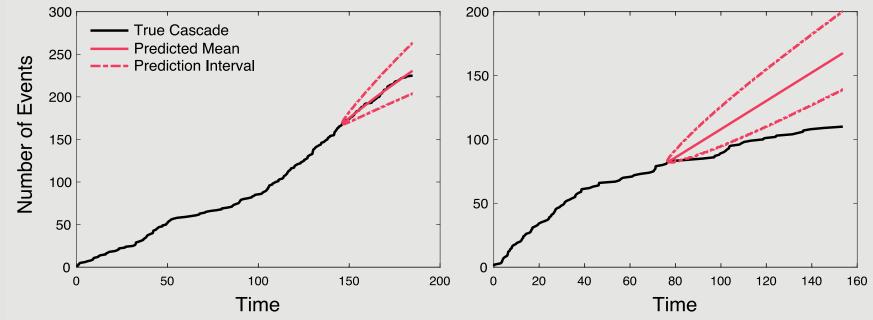


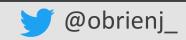


🈏 @obrienj_

Issues With the Model

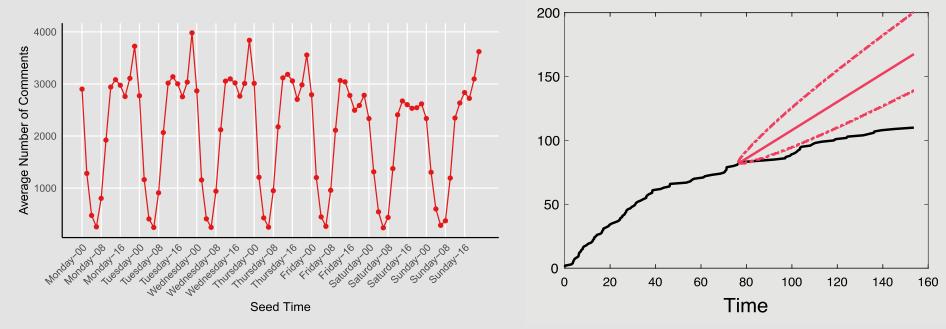
- We require enough activity in the observation period to provide accurate parameter estimation.
- In all of the above we have assumed constant background intensity so can capture the daily variation in the number of users present online.
- In spite of this, we can provide **useful predictions** regarding **future thread popularity** with results comparable to the current literature (without extensive numerical computations).





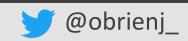
Issues With the Model

- We require enough activity in the observation period to provide accurate parameter estimation.
- In all of the above we have assumed constant background intensity so can capture the daily variation in the number of users present online.
- In spite of this, we can provide **useful predictions** regarding **future thread popularity** with results comparable to the current literature (without extensive numerical computations).

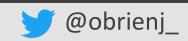




- Branching processes are extremely **powerful tools** in describing online information cascades.
- Developed a multi-type BP to capture the empirically observed cascades on multiple social media platforms – generalizing the 'competitioninduced-criticality' result.
- Can fully describe the **Hawkes process** allowing new results to be determined and allow **theoretical predictions** to be made for future cascade dynamics.
- Plenty of scope for further extensions...

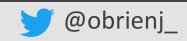


- Branching processes are extremely **powerful tools** in describing online information cascades.
- Developed a multi-type BP to capture the empirically observed cascades on multiple social media platforms – generalizing the 'competitioninduced-criticality' result.
- Can fully describe the **Hawkes process** allowing new results to be determined and allow **theoretical predictions** to be made for future cascade dynamics.
- Plenty of scope for further extensions...

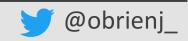


- Branching processes are extremely **powerful tools** in describing online information cascades.
- Developed a multi-type BP to capture the empirically observed cascades on multiple social media platforms – generalizing the 'competitioninduced-criticality' result.
- Can fully describe the **Hawkes process** allowing new results to be determined and allow **theoretical predictions** to be made for future cascade dynamics.

• Plenty of scope for further extensions...

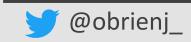


- Branching processes are extremely **powerful tools** in describing online information cascades.
- Developed a multi-type BP to capture the empirically observed cascades on multiple social media platforms – generalizing the 'competitioninduced-criticality' result.
- Can fully describe the **Hawkes process** allowing new results to be determined and allow **theoretical predictions** to be made for future cascade dynamics.
- Plenty of scope for further extensions...



- Branching processes are extremely powerful tools in describing online information cascades.
- Developed a multi-type BP to capture the empirically observed cascades on multiple social media platforms generalizing the 'competitioninduced-criticality' result.
- Can fully describe the **Hawkes process** allowing new results to be determined and allow **theoretical predictions** to be made for future cascade dynamics.
- Plenty of scope for further extensions...
- Don't prep a talk the day after a U.S. presidential election!!!





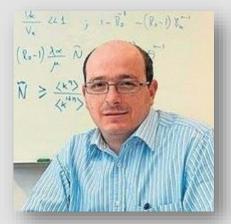
Collaborators & Thanks



James Gleeson (Uni. of Limerick)



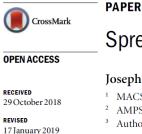
Ioannis Dassios (Uni. College Dublin) Alberto Aleta (ISI Foundation, Turin)



Yamir Moreno (BIFI, Uni. Of Zaragoza)

New Journal of Physics

The open access journal at the forefront of physics



Spreading of memes on multiplex networks Joseph D O'Brien^{1,3}, Joannis K Dassios², and James P Gleeson¹

MACSI, Department of Mathematics and Statistics, University of Limerick, Ireland
 AMPSAS, University College Dublin, Ireland

³ Author to whom any correspondence should be addressed.

PHYSICAL REVIEW E 101, 062311 (2020)

Quantifying uncertainty in a predictive model for popularity dynamics

Joseph D. O'Brien ⁽¹⁾, ¹ Alberto Aleta ⁽²⁾, ^{2,3} Yamir Moreno ⁽³⁾, ^{2,3,4} and James P. Gleeson ⁽³⁾ ¹MACSI, Department of Mathematics and Statistics, University of Limerick, Limerick V94 T9PX, Ireland ²Institute for Biocomputation and Physics of Complex Systems, University of Zaragoza, Zaragoza 50018, Spain ³ISI Foundation, 10126 Turin, Italy

⁴Department of Theoretical Physics, Faculty of Sciences, University of Zaragoza, Zaragoza 50009, Spain

(Received 26 January 2020; accepted 3 June 2020; published 22 June 2020)

Thank you for listening!

New J. Phys. 21 025001 (2019); Phys. Rev. E, 101.062311 (2020)

🅤 @obrienj_